Free software for the determination of positive and inverted S-curves for the response function of influential treatments or conditions with examples of crop yield versus soil salinity and depth of the water table.

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Abstract

The application of influential treatments often results in an S-shape response function of the goal factor. The postive S-function exhibits initially a slow increase of the goal factor (Y) with increasing values of the treatment (or conditional) variable (X), followed by a gradually increasing effect, then, beyond the inflection point, the increases diminish and finally approach an almost horizontal line meaning that the increase approximates a zero value. The inverted S-curve describes the opposite trend: first there is a slow decrease where after the decrease rate becomes larger until reaching the inflection point beyond which the decrease rate slows down and the curve finally approaches an almost horizontal stretch. The positive S-curve is ascending while the inverted (negative) one is descending. This report explains the mathematical background of the S-curve, the operation of the software and gives examples of various positive and negative cases in agriculture, like the crop response to soil salinity and depth of the water table.

Contents

1. Introduction

In literature the use of S-curves to describe the influence of an independent variable on the dependent variable is mainly related to business and are described only in general terms.
A more specific report was published by Steppuhn et al., 2005, who discussed the Gompertz, Weibull, and bi-exponential S-curves, but a description of the methods to solve these models was not given. [**Ref. 1**]. Also, they present only one simple data set and do not place the models in a wider context.

The free SegRegA software helps in determining the generalized logistic S-curve fitting given Y (the dependent variable) and X (the treatment or conditional) data [**Ref. 2**]. It solves both the positive (ascending) and negative (descending, inverted) S-curves. The mathematics are described in section 2, the operation in section 3, and examples in sections 4 and 5.

2. S-curve mathematics

The S-curve mathematics are explained in Figure 1, a screen print of the output of SegRegA concerning a study of the trend of the rainfall (the Y data) at the KNMI meteorological station in De Bilt, The Netherlands, over the years 1900 to 2020 (the X data). [**Ref. 3**].

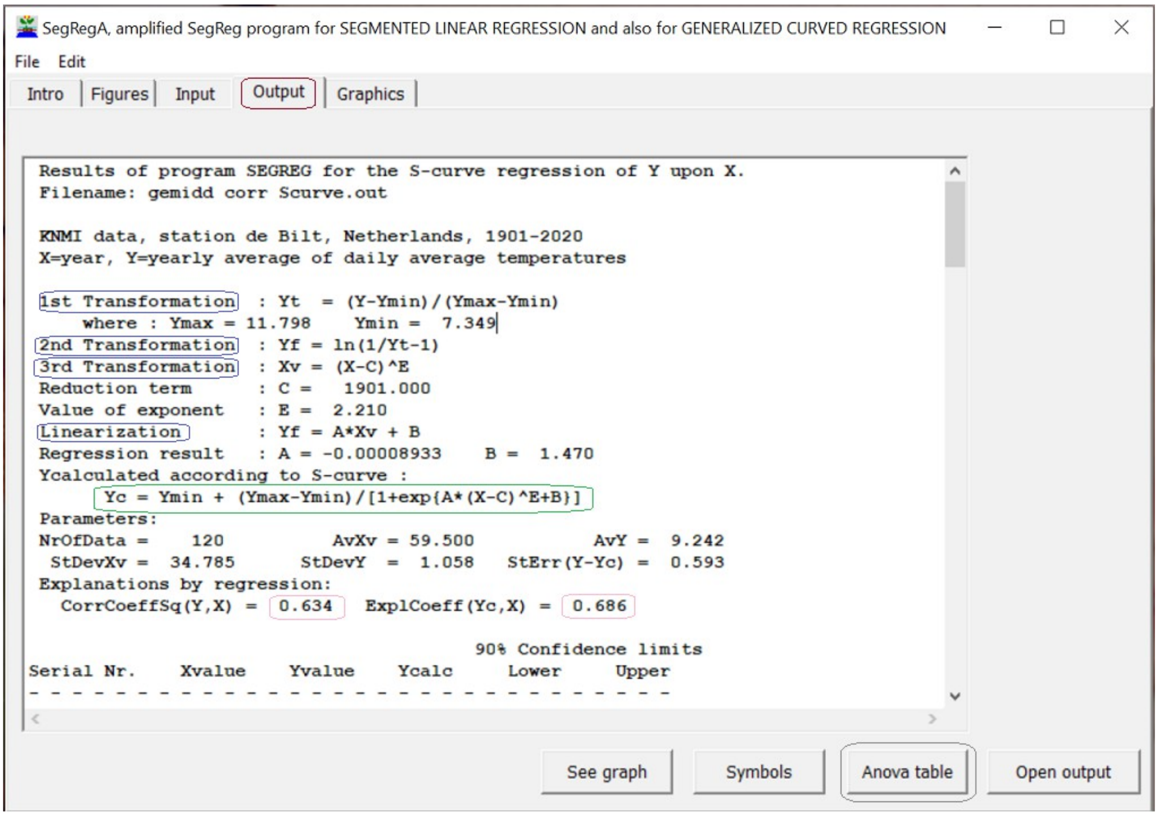

Figure 1. *The mathematics of the S-curve can be seen in the output sheet of SegRegA. There are 3 transformations leading to a linearization by which the A and B values can be found from a linear regression. The square value of the correlation coefficient, which is the explanation coefficient if a simple linear regression would have been done without transformations equals 0.634 (or 63.4%) while the transformations leading to the S-curve yield a coefficient of explanation of 0.686 (or 68.6%), which is slightly higher. The Anova table (Figure 2) can tell if the improvement is statistically significant.*

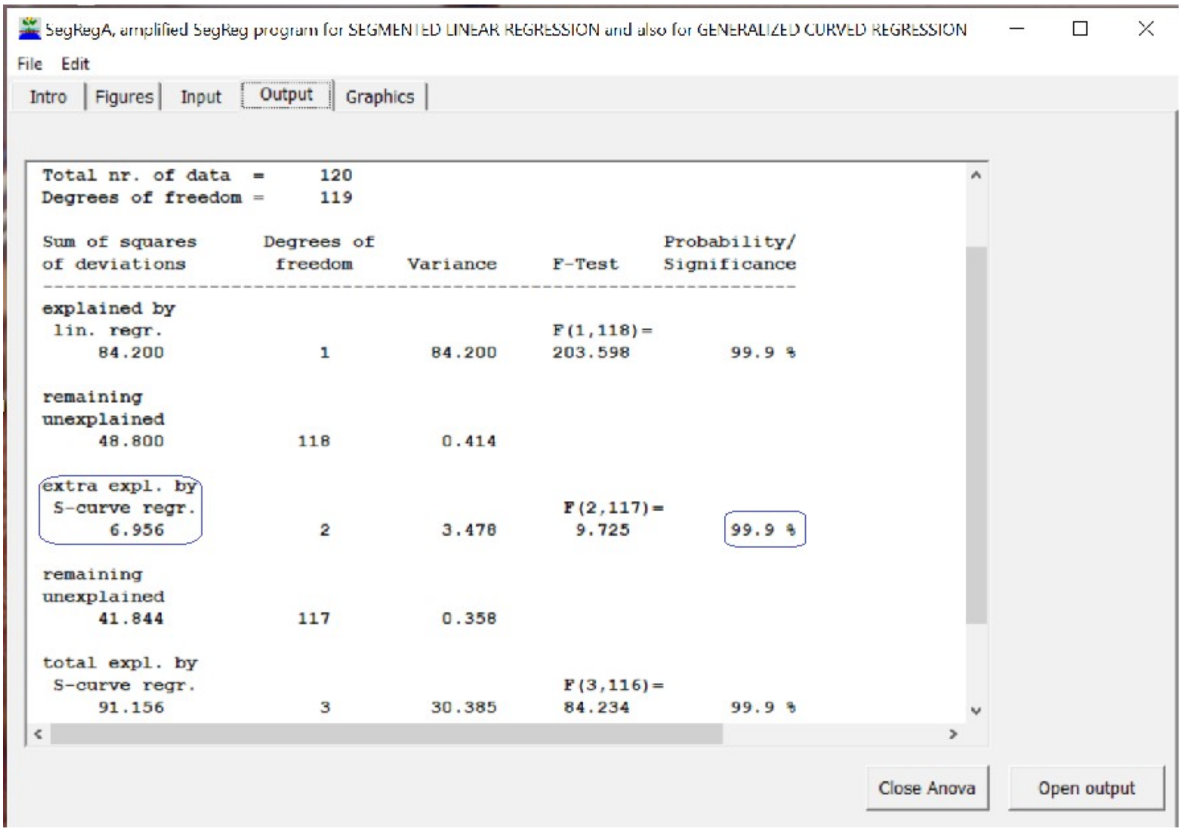

Figure 2. *Anova table for the case of Figure 1. It can be seen that the F-test for the extra explanation by the S-curve regression over a simple linear regression is highly significant.*

3. SegRegA software

In the following figure, the input options have been shown. After completing the operations and decisions, the "Save-Calculate" button can be clicked, so that the computations will be done and the output sheet will appear as shown in figure 1. In the following sections the output results of the examples employed will be discussed, making use of graphs that can be produced using the "See graph" button depicted at the bottom of figure 1.

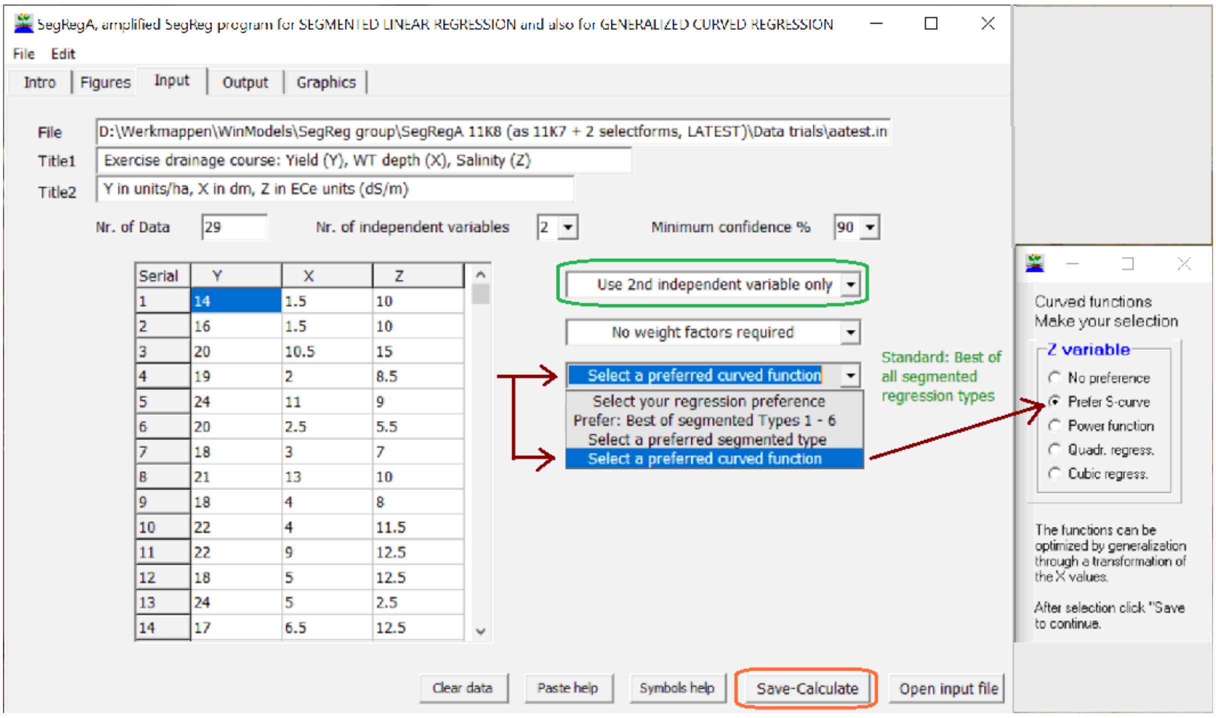

Figure 3. *The input menu of SegRegA. The input data have been pasted from Excel into the input table. The second independent variable (Z) has been selected for further analysis (green rectangle, the options are both variables, the first variable only or the second variable only). In the selection box, the group "curved functions" has been chosen while in the relevant decision box the preference for the "S-curve" has been fixed. Use "Save-Calculate"(red box) to continue.*

4. Crop yield decline under high soil salinity

In literature the are abundant reports about the salt tolerance of crops to soil salinity, triggered by Richards et al, 1954, of the US Soil Salinity laboratory, Riverside, California [**Ref. 4**]. The vast majority of the data were obtained from controlled experiments.

The paper on "Crop production and soil salinity" [**Ref. 5**] was almost the first to analyze data obtained in farmers' fields, .which show considerably more variations than the data obtained from the laboratory**.** Initially the analysis was done using segmented envelopes. Later, an addendum was added to their article in which the PartReg software [**Ref. 6**] was used to detect horizontal stretches in the relation between crop production and soil salinity so that the point beyond which the yields start to decline could be defined. This point is called tolerance level.

The article on "Crop tolerance to soil salinity" [**Ref. 7**] gives an overview of methods used in literature to determine the tolerance level, including the well known Maas-Hofmann model and the less used van Genuchten-Gupta model, which is a kind of S-curve. In general, S-curves have not been applied often because these methods do not produce a clear cut tolerance level. Also, until recently, no software was available for S-curve determination so that wide spread application did not occur owing the mathematical complexity.

The Maas-Hoffman model [**Ref. 8**], which shows a horizontal stretch of the yield-soil salinity relation at the lower values of the soil salinity followed by a descending straight line has been incorporated in the SegRegA [Ref. 2] software under the selection "segmented type" (figure 3) under Type 3 (out of 6 types).

As this article is mainly about S-curves, the next figure shows and example concerning the yield of barley versus soil salinity. The data stem from [Ref. 5] and have been analyzed with SegRegA. The graph has been produced the "See graph" button shown in figure 1.

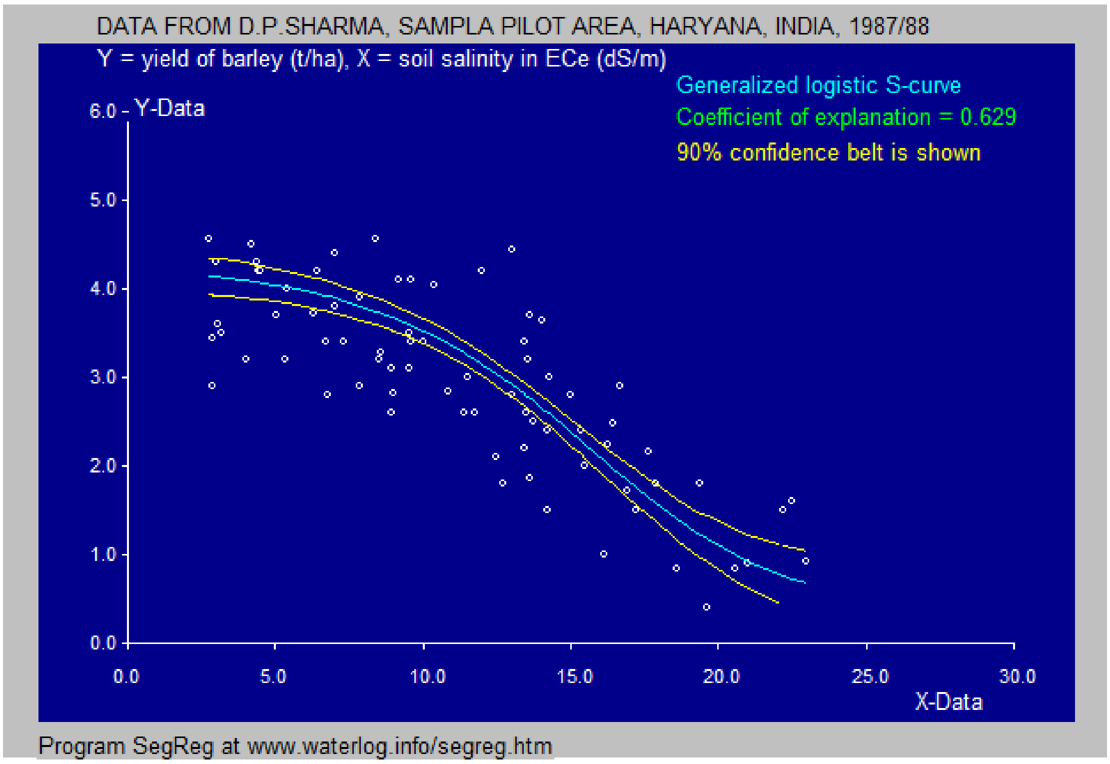

Figure 4. *Negative S-curve with descending trend of dependent variable Y, being the crop yield of barley, with increasing values of the independent, influential, variable X, representing the soil salinity. The data were measured in a cultivated field, under less controlled conditions compared to the situation in a laboratory, and therefore the variation is large. Yet the coefficient of explanation is relatively high and the Anova table proves a statistically high significance, like in figure 2.*

Instead of using SegRegA with the S-curve choice, One could also use the PartReg program [**Ref. 9**] to detect horizontal stretches in (Y,X) data sets. Figure 5 shows a Z-type relation, found with PartReg, using the same data as in figure 4. This relation shows similarity with the S-curve

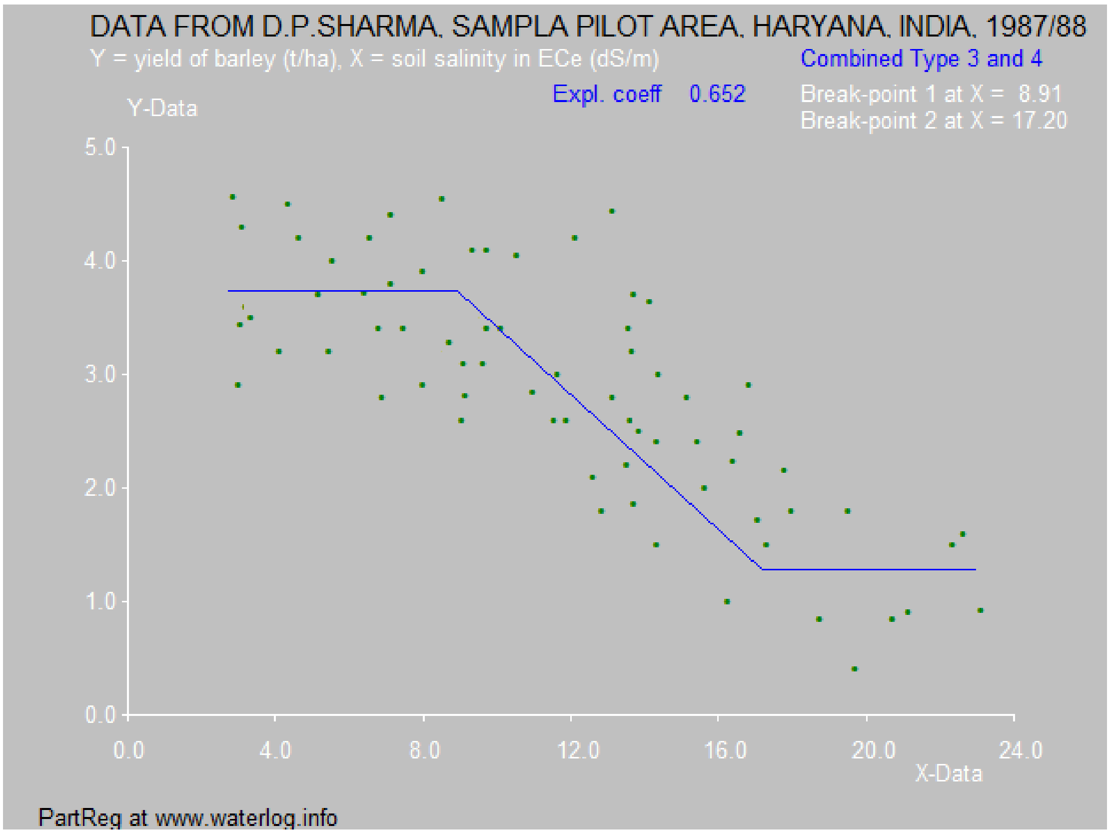

Figure 5. Result of the application of the PartReg software to the same data as used in figure 4 that depicts an S-curve. The advantage of the Z curve is that it marks the tolerance level of barley to soil salinity at ECe = 8.9 (the electric conductivity of an extract of a saturated soil sample), which indicates that barley is quite salt tolerant.

Another example of an S-curve is given in the next graph for the yield of potato versus soil salinity. The data are obtained from the publication "Potato variety 927" [**Ref. 10**].

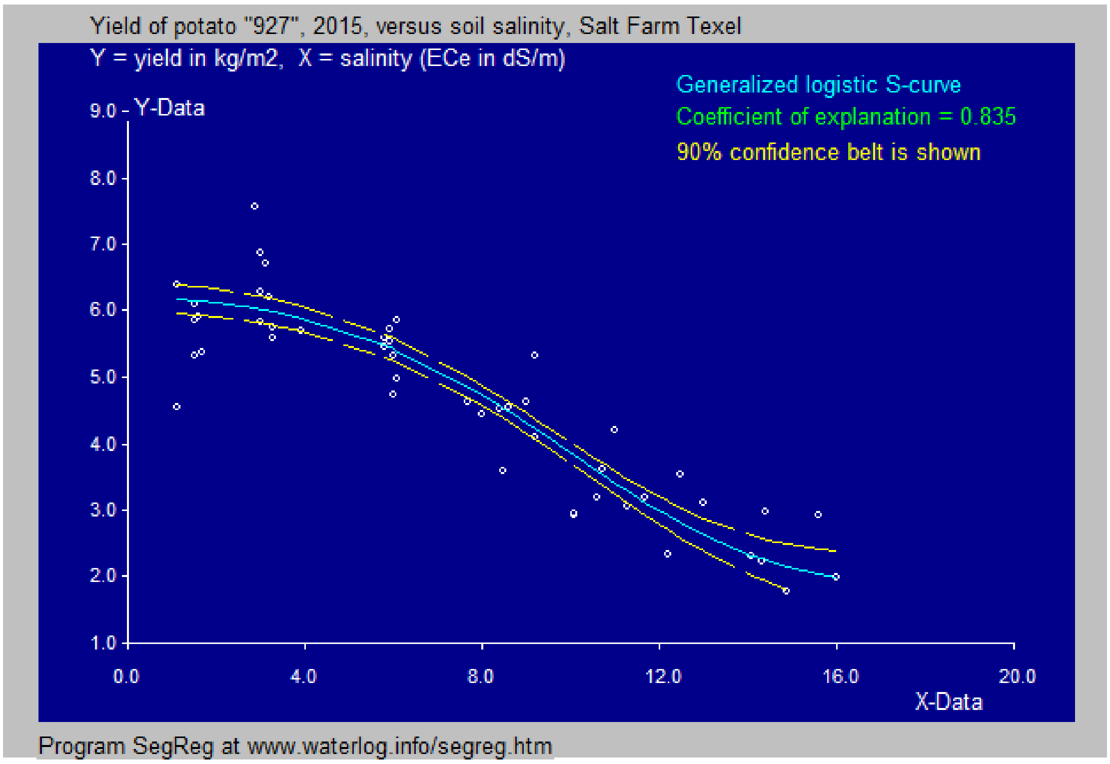

Figure 6. *Yield of potato variety "927" versus soil salinity with a generalized logistic S-curve fitted to the data. The coefficient of explanation is quite high, despite the variations one can expect under field conditions.*

Instead of applying the S-curve, one could also use a cubic regression as can be seen under the choices shown in figure 1. The result employing the same data as in the previous figure 6, is depicted below in figure 7.

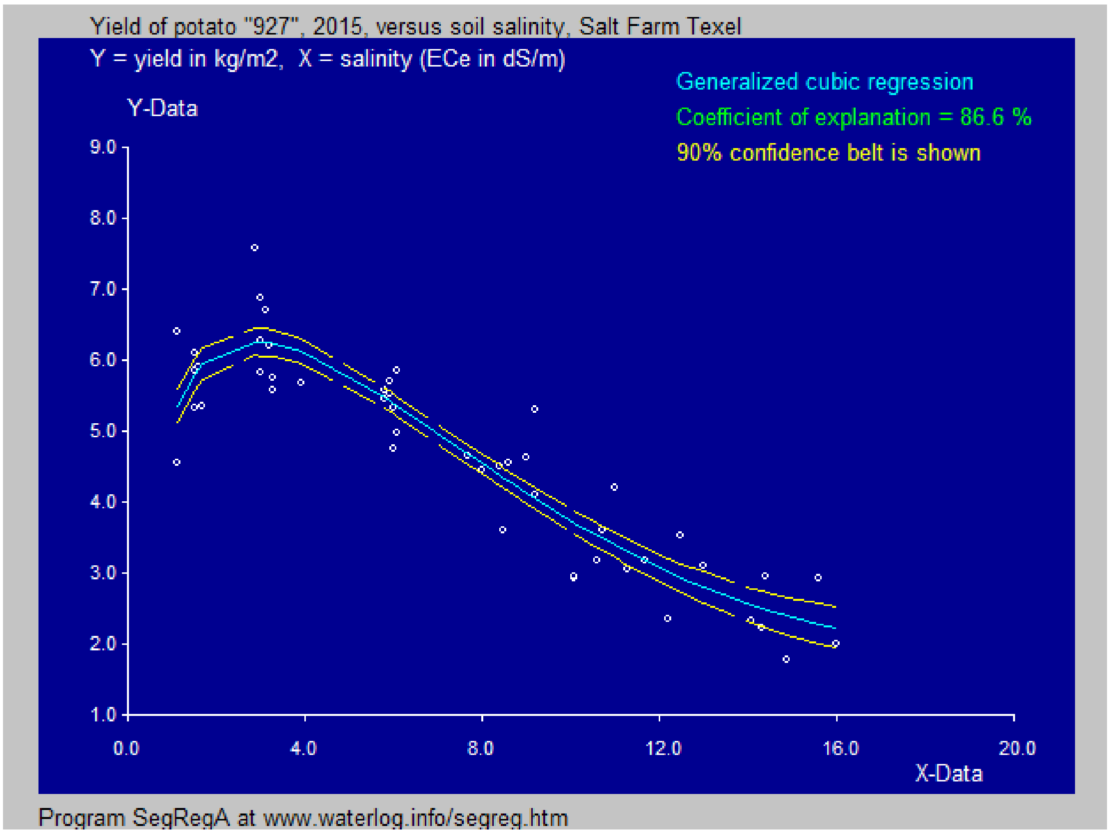

***Figure** 7. Cubic regression using the "927" data of the Salt Farm Texel as in figure 9. The regression equation is Y = 0.537*Z^3 – 4.70*Z^2 + 11.2*Z – 1.84, where Z = X^0.49, the exponent 0.49 effectuating a generalization of the cubic regression (in other words the X values are raised to the power 0.49 before the cubic regression is done, this to increase the goodness of fit). As the exponent is smaller than 1, the order of the equation is less than 3 as would be the case in a true cubic(third degree) equation. The coefficient of explanation (also called Rsquared) equals 86.6%, which is quite high.*

Compared to figure 6, figure 7 shows an initial wave at low salinity, indicating that the crop needs at least some salts for its development. Beyond the salinity ECe = 4, the shape of the cubic regression curve in figure 7 is practically the same as that of the S-curve in figure 7.

5. Crop yield decline under shallow water table

The articles on “drainage criteria” [**Ref. 11**], “effects of drainage on agriculture” [**Ref. 12**], and “effects of drainage on agriculture” [**Ref. 13**] and “crop yield and depth of water table” [**Ref. 14**] attempts were made to collect information on the relation between crop yield and depth of the water table, but the information was scarce. It was even more difficult to find a relation to which the S-curve could be fitted.

Figure 8 demonstrates an S-curve fitted to the relation between yield of sugar cane and seasonal average depth of the water in Australia. Data found in the publication of Rudd. and Chardon, 1977 [**Ref. 15**].

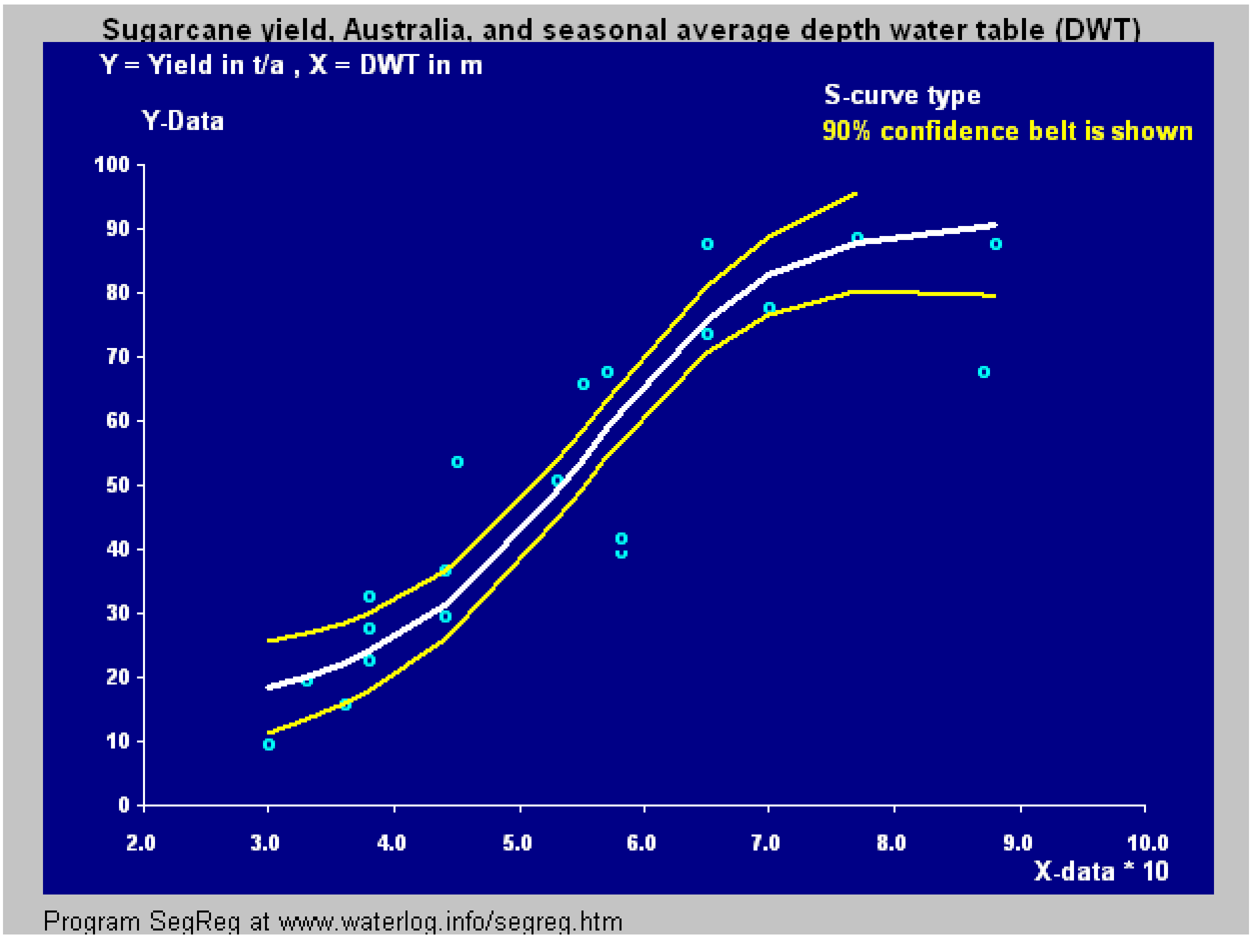

Figure 8. *An S-curve fitted to data on yield of sugar cane and depth of the water table using SegRegA. Contrary to the soil salinity cases, the water table case shows a positive (ascending) S-curve. At shallow depths of the water table, the yield is low. From depth 0.5 m to 0.7 m the yield increase sharply. After that, the slope of the S-curve flattens out. The conclusion could be that, to obtain good yields, the water table should be at least 0.7 m deep, but preferably 0.9 m or more. It is not worth to spend more on a costly drainage system to realize depths of 1.0 m or more, as that does not lead to a higher yield any more.*

6. Conlusions

The SegRegA software is helpful for the fiiting of S-curves to (Y,X) data sets. It uses the principles of the logistic probability distribution. In addition it uses a transformation of the X data with an exponent that can be larger or smaller then 1 (in figure 1 its value is 2.2). This enhances the versatility of the method.

Many functions of a variable Y dependent on the value of the independent, influential, X data that may represent a treatment or condition, can be simulated through S-curves. Successful examples were given of crop yield (Y) and soil soil salinity or depth of the groundwater table as X variable. The S-curve technique is probably not only useful in agricultural science, but also in medical and other sciences

7. References

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