RAINFALL-RUNOFF RELATIONS IN A SMALL CULTIVATED VALLEY IN SIERRA LEONE

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CONTENTS

INTRODUCTION

The increased demand for rice, as a staple food in Sierra Leone, requires more intensive use of the hydromorphic soils for rice production.

Sposored by the Netherlands` Directorate General for International Cooperation and in collaboration with the International Institute for Land Reclamation and Improvement (ILRI, Wageningen) and the Land and Water Development Division (LWDD) in Sierra Leone, the International Institute of Tropical Agriculture (IITA, Nigeria) initiated the Wetland Utilization Research Project. One of its objectives is to develop low input water management technologies, that will enable smallholders to cultivate the small valleys of West Africa more intensively.

This report covers the research period of October 1985 to January 1987. During this period the rainfall-runoff relations in the Rogbom catchment were monitored. This catchment is located four miles east of Makeni, in the centre of Sierra Leone. The catchment's size is 116ha. The area of the valley is 18 ha (15%), the upland covers 80 ha (70%) and the rest of the area (18 ha, 15%) is fringeland.

This is the fifteenth report of the Wetland Utilization

Research Project and presents the analysis of the monitored rainfall and runoff hydrographs, making use of the 'De Zeeuw' model.

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Special mention is made here of the support of Ir. Roland Oosterbaan of the Institute for Land Reclamation and Improvement in Wageningen, the Netherlands.

NOTATIONS

The symbols that are frequently used and not specified everywhere are summarized below.

Symbol	Description	Dimension
a	reaction factor	1/day
af	fast reaction factor	1/day
ar	surface runoff reaction factor	1/day
as	slow reaction factor	1/day
p	effective rainfall	mm/hr
q	specific discharge	mm/hr
q(0)	initial specific discharge	mm/hr

q(1)	specific discharge at t=1	mm/hr
qf	discharge of fast reservoir	mm/hr
qr	discharge of surface reservoir	mm/hr
qs	discharge of slow reservoir	mm/hr
f	distribution coefficient for the fast reservoir	1
r	distribution coefficient for the surface reservoir	1
s	distribution coefficient for the slow reservoir	1
t	time	hr
U	objective function (squared deviation of calculated and measured rainfall)	
S	standard deviation derived from U	mm sq mm

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1. HYDROLOGICAL MODELLING

In recent years many different procedures have been proposed to solve the complex physical relationships that exists within a catchment during the rainfall-runoff phase of the hydrological cycle. These relationships include the interception by vegetation, infiltration into the soil surface, the evapotranspiration and the dynamics of overland, channel and groundwater flow.

Many analytical models have been developed which ignore these complex physical processes and simply attempt in an experimental way to relate runoff to rainfall. In its crudest form this could be by way of a simple correlation. But usually some mathematical functions are applied to describe the variations of runoff with the rainfall. These models are usually termed 'Black Box' models. Despite the functions, they make no attempt to simulate any of the individual hydrological processes. Hence the parameters of these models have very little, if any, physical meaning.

Still it would be very useful if the parameters of the black box model could be related to measurable physical characteristics, so that a grey box is obtained. In search of such a model, hydrologists have attempted to simulate specifically each of the hydrological components of the catchment (e.g. interception, infiltration, groundwater flow, evapotranspiration and surface water flow). The advent of high-speed computers, enabling the handling of very large numbers of calculations, has led to the development of a large number of these 'conceptual' models in recent years. Some of these are very complex, for example the SWATRE model in the Netherlands and the Stanford model in the UK, whereas others have simplicity as their prime aim.

Pitmann (1978) concluded that, when optimum values of the parameters have been found, simple models are as efficient as more elaborate ones. In view of this it was decided to use the simple 'de Zeeuw' model.

2. THE 'DE ZEEUW' MODEL

2.1. Introduction

The 'de Zeeuw' model characterizes a catchment by reaction factors, which can be obtained from the analysis of hydrographs, because "... the discharge hydrograph of an area necessarily shows the hydrologically characteristic properties of that area and will yield the parameters of the model." (J.W. de Zeeuw, 1973).

According to de Zeeuw the catchment can be divided into two or more reservoirs, for example a surface and a subsurface reservoir, each having its own reaction factor. This allows certain parts of the catchment to belong to the fast reacting reservoir during wet periods and to the slower reacting reservoir during dry periods.

Although the model originally was developed for flat areas (polders), it gives good results for sloping catchments with natural drainage.

The basic idea of this model lies in the assumption that the discharge is proportional to the waterheight above the drainage level.

2.2. Theory of the linear reservoir

The most simple drainage model is that of a linear reservoir. We assume a reservoir, with surface area A, that can be emptied by a capillary tube. If the tube is not too large, the flow through it is laminar and the discharge Q is proportional to the head h:

$$Q = c.h$$

Per unit of area, the speciffic discharge q is proportional to h, thus:

$$q = Q/A = a.h$$

If at time t=0 a rainfall p starts, the waterbalance during the time dt can be written as a first order, linear, differential equation:

$$(p-a.h)dt = dh$$

After integration follows:

$$p-a.h = C.\exp(-a.t)$$

where C is a constant, and exp(-a.t) means: the natural base e (approx. 2.71) raised to the power -a.t.

At the time t=0, the initial discharge q equals q(0), so that:

$$C = p-q(0)$$

This leads to:

$$q = q(0).\exp(-a.t) + p[1-\exp(-a.t)]$$

which is called the respons of a linear reservoir.

2.3. Application with several reservoirs

In this section, the elements needed for the reconstruction of the observed discharge will be discussed.

The first element is the reaction factor (a) of the respons equation of section 2.2. To find this element, the observed discharge hydrograph is plotted on semi-log paper. From the slope of the tail-end of the plotted hydrograph, the (smallest) reaction factor is found. During a rainless period, the respons equation reduces to:

$$q(2) = q(1).\exp\{-a.[t(2)-t(1)]\}$$

or:

$$a = 2.30\{\log[q(2)]-\log[q(1)]\}/[t(2)-t(1)]$$

note: $\log[\exp(1)] = 1/2.30$

After extending the straight tail-end line to the left, the hydrograph has been divided into two parts. The area below the straight line represents the discharge of the slow reservoir (s) and the area between the straight line and the hydrograph represents that of the fast reservoir (f).

Replotting the vertical distances between these two lines on semi-log paper results in another straight line, representing the contribution of the fast reservoir to the discharge. The slope of this straight line gives the reaction factor of this reservoir. This procedure can be continued till all reservoirs, and their reaction factors, are known.

In the model, the rainfall must be distributed over the different reservoirs. This is done by means of distribution coefficients (the second element). Of course, the sum of all distribution coefficients equals one. Like the reaction factors, the distribution coefficients are also derived from the observed hydrograph. This is done by comparing the total amounts of runoff obtained after the separation of the reservoirs.

For a catchment with surface runoff and two groundwater reservoirs (fast and slow), the model reads as follows:

$$qr' = r[qr(0).\exp(-ar.t) + p\{1-\exp(-ar.t)\}]$$
$$qs' = s[qs(0).\exp(-as.t) + p\{1-\exp(-as.t)\}]$$
$$qf' = f[qf(0).\exp(-af.t) + p\{1-\exp(-af.t)\}]$$

where f, r and s are the distribution coefficients, and q(0) is the initial total discharge (or runoff) rate of the catchment. The respons of this system of three parallel linear reservoirs reads:

$$q = qr' + qs' + qf'$$

According to de Zeeuw the order of magnitude of the reaction factors in the Netherlands is as follows:

$ar = 1.0$ to 3.0
$af = 0.3$ to 0.7
$as = 0.1$ to 0.001

3. THE HYDROGRAPH ANALYSIS

This chapter describes the analysis of hydrographs, obtained from the Rogbom Catchment during the period June to September 1986. Not all observed hydrographs were considered to be suitable for the analysis. Three conditions had to be met:

- an instantanious rainfall, to assure a distinct runoff peak;
- a dry spell of at least 2 days after the rainfall, to assure that the fast reservoirs will be emptied;
- a dry spell before the rainfall to assure that the fast reservoirs do not contribute to the runoff q(0) at the start of the hydrograph analysis.

These conditions were met by 15 hydrographs (2 in June, 8 in July, 4 in August and 1 in September).

3.1 Finding the effective rainfall

The term effective rainfall has been interpreted differently by specialists in different fields and even by different workers in the same field.

A farmer considers that effective rainfall is the quantity which is useful in raising crops planted on his soil, under his management.

For the study of rainfall - runoff relations, the effective rainfall is taken as the part of the rainfall that is transformed into runoff.

For Sierra Leone, no information on effective rainfall could be traced. Therefore, it was found neccessary to make use of the waterbalance of the Rogbom valley obtained from the research during the 1986 growing period. For this report, the effective rainfall has been taken as the total amount of rainfall minus the evaporation (see Table 3.1).

Table 3.1: Waterbalance per month.

month	rainfall	evaporation	eff.rainfall
May	351.4	115.7	235.7
June	369.5	87.8	281.7
July	596.6	71.7	524.9
August	539.1	63.1	478.0
Sept.	545.9	71.8	474.1
Oct.	330.6	90.0	240.6

As can be seen from Table 3.1, the effective rainfall is about 80% of the measured rainfall. It was considered to be acceptable, to define the effective rainfall as 80% of the total amount of measured rainfall. In other words, the parameter p in the 'De Zeeuw' model equals, in this report, 80% of the measured rainfall

3.2. Finding the reaction factors

To find the reaction factors, the observed hydrographs were plotted on semi-log paper. The slope of the tail-end gives the smallest reaction factor (as). The replotted differences between the observed hydrograph and the extended tail-end, gives the fast reaction factor (af). See annex A for the graphical presentation. The results are summarized below.

Table 3.2: Reaction factors of 15 observed hydrographs.

date	as	af
22-06	0.18	12.1
26-06	0.18	8.2
03-07	0.22	5.0
07-07	0.26	7.3
09-07	0.18	7.5
14-07	0.10	6.1
16-07	0.13	7.3
21-07	0.15	6.6
25-07	0.07	5.8
28-07	0.15	5.8
01-08	0.14	7.4
06-08	0.07	8.5
21-08	0.06	4.9
25-08	0.06	5.7
07-09	0.09	6.9

As can be seen from annex A, it is not quite clear whether there are one or more fast reservoirs. For simplicity sake, it is assumed that there is only one. In doing so, the respons of the system of parallel reservoirs has been reduced to:

$$q = qs + qf$$

The physical meaning of the two reservoirs is not clear: it may be that the fast reservoir accounts for the total surface runoff and part of the groundwater flow, but it may also be that the slow reservoir accounts for the total groundwater flow and part of the surface runoff.

3.3. Finding the distribution coefficients

As explained in chapter 3.2, the respons of the system has been reduced to q = qs + qf. From the analysis of the 15 observed hydrographs it appeared that, throughout the analysis period, the distribution coefficients of the rainfall over the reservoirs are s=0.7 and f=0.3 when the discharge from the fast reservoir, qf, is larger than that of the slow reservoir, qs. When qf becomes smaller than qs, the distribution coefficients are s=1.0 and f=0.0.

3.4 Adjustment of the reaction factors

The reconstruction of the runoff hydrographs with the reaction factors calculated was not very successful. Therefore, the reaction factors were recalculated as follows:

- select for each measured hydrograph a value of af, so that, after reconstruction, the peaks of the measured and reconstructed hydrograph show close correspondence;
- select for each measured hydrograph a value of as, so that, after reconstruction, the tail-ends of the measured and reconstructed hydrographs show close correspondence.

The results of this procedure, as described in the next chapter, are given in Annex B and summarized in table 3.3.

Table 3.3: Adjusted reaction factors of 15 observed hydrographs.

date	as	af
22-06	0.10	6.0
26-06	0.18	6.0
03-07	0.18	6.0
07-07	0.18	8.0
09-07	0.18	6.0
14-07	0.15	6.2
16-07	0.10	5.0
21-07	0.18	10.0
25-07	0.07	5.2
28-07	0.15	5.8
01-08	0.08	8.0
06-08	0.07	8.0
21-08	0.07	6.9
25-08	0.07	5.7
07-09	0.09	7.8

4. RECONSTRUCTING THE RUNOFF

Now that the effective rainfall, the reaction factors and the distribution coefficients are known, the runoff hydrographs can be reconstructed, using the measured rainfall and the initial discharge. As stated in chapter 3, there was a distinct dry period before every analysed hydrograph. As this dry period lasted approx. two days, it is save to assume that the initial discharge from the fast reservoir, qf(0), was zero. This means that the initial discharge of the slow reservoir equals the measured discharge.

During the reconstruction of the June and July hydrographs, that is at the beginning of the season when the valleys are not yet ponded, three of the fifteen hydrographs were hard to reconstruct. These three hydrographs had a distinct rainfall pattern. Approx. 80-90% of the rainfall fell during the first hour of the shower, whereas for the other hydrographs this was less.

As can be seen in annex B, the majority of the hydrographs have their peak discharges appr. 3 hours after the start of the rainfall, the so called "time to peak". This has led to the following modification: during the period that the valley is not yet ponded (the period before the second week of July) the rainfall should be equally devided over a three hours time span, if more than 70% of the rainfall occurs during the first hour of the shower. This modification (although it is not more than just an empirical adjustment) has led to fairly good results.

The fifteen hydrographs were reconstructed, using a computer program written in Fortran IV. The program is listed in annex C. The results of the reconstruction can be found in annex B. They are summarized in table 4.1.

To test the results, an objective function (Diskin, 1977) was introduced in annex B:

U = [y(i)-x(i)]Sq

in which the y(i) are the observed values of the runoff for a specified time interval, x(i) are the values generated by the model for the same interval, and Sq (squared) represents the raising to the second power. The U value is converted into a stadard deviation as follows:

S = Rt(SumU/n)

where Rt stands for the square root, SumU for the

summation of U over the intervals during the period of analysis and n for the number of intervals.

Further, annex B uses a Q(*) value, which is the sum of the observed discharges (in mm) over the intervals minus the sum of the generated discharges (in mm) over the same intervals during the period of reconstruction:

$$Q(*) = \text{Sum } Q(obs) - \text{Sum } Q(gen)$$

Table 4.1: Results from the reconstruction.

	discharges in mm during 30 hrs			peak discharges in mm/hour		
date	Qcalc	Qobs	Qc/Qo	Pcalc	Pobs	Pc/Po
22/6	15.4	14.5	1.06	1.87	1.65	1.13
26/6	11.7	11.3	1.04	1.07	0.98	1.09
13/7	10.5	10.6	0.99	0.91	0.75	1.21
7/7	26.9	27.6	0.97	2.03	2.32	0.88
9/7	39.7	41.0	0.97	3.15	3.08	1.02
14/7	31.9	32.3	0.99	2.24	2.10	1.07
16/7	36.3	39.0	0.93	3.55	3.35	1.06
21/7	31.3	30.6	1.02	2.22	2.25	0.99
25/7	32.1	31.5	1.02	1.51	1.59	0.95
28/7	35.8	36.0	0.99	2.73	2.68	1.02
1/8	46.5	46.9	0.99	3.38	3.11	1.09
6/8	34.8	35.9	0.97	1.71	1.76	0.97
21/8	29.4	30.6	0.96	1.64	1.72	0.95
25/8	32.9	33.5	0.98	1.65	1.76	0.94
7/9	50.4	52.5	0.96	3.30	3.39	0.97
mean			0.99			1.02
st.dev			0.034			0.086

The standard deviations of the ratios is relatively small, because the parameters used in the model to generate the hydrographs were obtained from the same observed hydrographs.

5. PREDICTION

In chapter 4, fifteen hydrographs were reconstructed after analizing the observed hydrographs. In this chapter three runoff hydrographs will be reconstructed for 3 rainstorms that have not yet been analized. Also the runoff during two decades will be reconstructed.

The results of this reconstruction will give insight in the predictive value of the modified "De Zeeuw" model.

Of the 3 rainstorms, the first occurred on 20th June (this is before the analized period); the second on 13th July (in the middle of the analized period) and the third one on 17th September (that is after the analized period).

Table 3.3 shows that the fast reaction factor is more or less a constant during the period of June to September 1986. For the prediction, the fast reaction factor (af) is assumed to be 6.0. The same table shows that the slow reaction factor (as) varies during the analyzed period. The factor tends to be smaller towards the end of the period. For the prediction, the slow reaction factor is assumed to be 0.16 till 15th July and 0.10 from 15th July till the end of the period.

The hourly rainfalls, used as input for the prediction, is shown in table 5.1. (As the valley was ponded on 17 September, there was no need to modify the rainfall).

Table 5.1: Hourly rainfall in mm.

time interval	20-06-86	13-07-86	17-09-86
0 - 1	12.8	4.0	17.4
1 - 2	6.2	13.5	54.6
2 - 3	4.8	9.2	6.0
3 - 4	2.8	3.3	0
4 - 5	0	6.2	0
5 - 6	0	3.6	0
6 - 7	0	0	0
7 - 8	0	1.7	0
8 - 9	0	0	0
9 - 10	0	0	0
10 - 11	0	6.0	0
11 - 12	0	3.1	0
12 - 13	0	1.0	0
13 - 30	0	0	0

The input of the program is listed in annex D, the output in annex E. The output results are summarized in table 5.2.

Table 5.2: Results of three predicted hydrographs.
(values in mm)

Rainfall date	20-06	13-07	17-09
Sum of the eff. rainfall	26.6	51.6	78.0
Sum of the obs. runoff	11.5	36.9	50.7
Sum of the gen. runoff	12.6	35.4	52.8
Stand. Dev. (S)	0.045	0.11	0.12

The results of these three generated hydrographs are also shown in fig. 5.1, 5.2 and 5.3 .

From the three figures and annex E, it may be concluded that the assumptions discussed in chapter 3 and the assumption that the fast reaction factor equals 6.0 during the whole rainy period, while the slow reaction factor is 0.16 till 15th July and 0.10 after 15th July, lead to fairly accurate generated hydrographs.

The input for the two decades is listed in annex F, while the output is listed in annex G. The output results are summarized in table 5.3.

Table 5.3.: Results of two predicted decade hydrographs. (Sums in mm.)

Decade	7-17 July	17-27 Aug
Sum of the eff. rainfall	223.1	128.8
Sum of the obs. runoff	229.3	217.9
Sum of the gen. runoff	196.7	191.3
Objective function	12.0	5.2

From annex G it can be seen that the 'De Zeeuw' model used for decades tends to describe fairly well the peak discharges, but it underestimates the tail-end discharges. Consequently it underestimates the initial discharges for generating a new rainstorm. Nevertheless, the predictive value of the 'De Zeeuw' model for the Rogbom Catchment is sufficiently accurate (thanks to the small value of the standard deviation) to be a basis for further water management improvement in this area.