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RAINFALL-RUNOFF RELATIONS OF A SMALL VALLEY ASSESSED WITH A NON-LINEAR RESERVOIR MODEL

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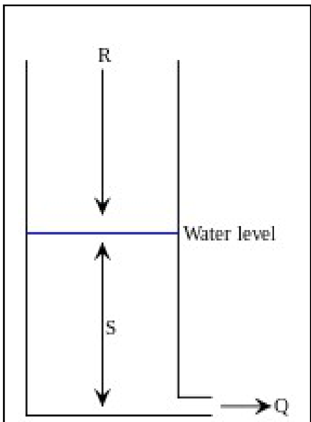
Abstract: - A non-linear reservoir model is used to analyze the rainfall-runoff relations in a small valley (a watershed, an hydrological catchment), in Sierra Leone. The concept of a linear reservoir, which uses a constant reaction factor, for use in hydrological modeling is well known but often not effective. Non-linear reservoirs, having reaction factors that depend on the water storage, are less frequently applied but they have more promise. One may use reaction factors that are a linear function of the storage, which implies that the reservoir reacts quicker to rainfall under wet than under dry conditions. The reaction factor could also be a quadratic function of the storage so that the discharge increases progressively with increasing water storage. The characteristic functions of the reaction factors of the catchments are first found by calibrating part of the data, and thereafter they are verified with the remaining data. The calibrations are done with a high precision. In this case, the verification was complicated by the fact that the valley bottom was used for rice cultivation and that the farmers interfered in the natural runoff process so that the reservoir characteristics changed in time. Yet, the non-linear reservoir model could be verified reasonably well.

Keywords : rainfall-runoff relations, small valley, rice cropping, non-linear reservoir model

1 Introduction, reservoir models

The linear reservoir is described by D.A.Kraijenhof van de Leur [Ref. 1] and its principles are given in figure 1.



R = rainfall {dimension L/T}

S = storage {dimension L}

Q = discharge, runoff {dimension L/T}

Fig. 1. The concept of a linear reservoir.

For the linear reservoir the following equations hold:

Reservoir function:

$$Q = \alpha . S \qquad \text{(Eq. 1)}$$

where α = reaction factor {1/T}

Differentiating S to time T gives

$$dS/dT = d(Q/\alpha)/dT = R - Q \qquad \text{(Eq. 2)}$$

Integrating Eq. 2 with limits Q_1, Q_2, T_1 and T_2 yields:

$$Q_2 = Q_1 \exp\{-\alpha (T_2 - T_1)\} + R\,[1 - \exp\{-\alpha (T_2 - T_1)\}] \qquad \text{(Eq. 3)}$$

where Q_2 and Q_1 are Q at time T_1 and T_1 respectively.

With Equation 3 the discharge Q_2 can be calculated from R, Q_1, α, and the time difference.

The well known instantaneous unit hydrograph (IUH), which is the runoff hydrograph resulting from a unit rainfall over a unit hydrograph when the initial runoff is zero (Q1 =0) can be found from Eq.2 as $Qu = 1 - e^{-\alpha}$.

An example of the application of the IUH method [Ref. 2] is given in figure 2.

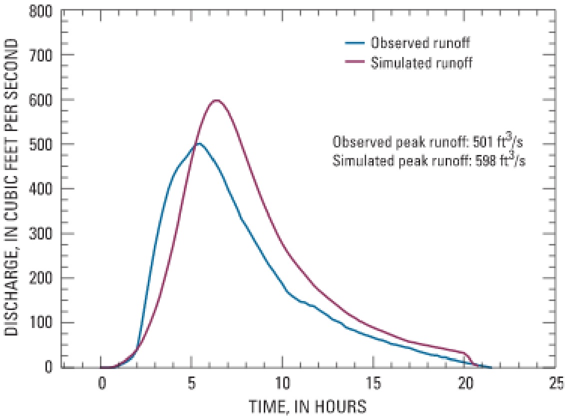


Fig. 2. Simulation by means of the IUH method of discharge runoff at Mallard Creek near Harrisburg, North Carolina, for the storm of December 12, 1996 [Ref. 2].

The reservoir model has similarity with the IUH method, but is has the advantage that it uses relatively simple mathematical principles, whereas the derivation and application of the IUH method is rather cumbersome, especially its calibration

When R=0 (no rainfall), equation 3 reduces to

$$Q_2 = Q_1 \exp(\{-\alpha\,(T_2-T_1)\,\} \qquad \text{(Eq. 4)}$$

This equation gives the possibility to determine, during a dry spell, α from Q_1, Q_2 and the time difference:

$$\alpha = -\ln\,(Q_2/Q_1)/(T_2-T_1) \qquad \text{(Eq. 5)}$$

This concept is often too simple to characterize the watershed as its reaction factor is usually more complicated. Therefore Nash [Ref. 3] employed a cascade of linear reservoirs, one reservoir emptying into the next, while Kraijenhoff [Ref. 1] used a number of parallel reservoirs over which the rainfall is distributed in some proportion, while the reservoirs joined their discharge.

In hydrology, the concept of non-linear reservoirs has seldom been applied. Instead of a reservoir with a constant reaction factor, one could employ a non-linear reservoir with a reaction factor that changes linearly with storage (figure 3) instead of being a constant, thus avoiding the difficulty of dealing with a series of reservoirs.

Equations 1, 2, and 3 now change into respectively

$$Q = (B.Q + C).S \qquad \text{(Eq. 6)}$$

$$dS/dt = R-(B.Q+C).S$$
$$= R - B.Q.S + C.S \qquad \text{(Eq. 7)}$$

$$Q_2 = Q_1 \exp\{-(B.Q_1+C).(T_2-T_1)\} +$$
$$R[1-\exp\{-(B.Q_1+C).(T_2-T_1)\} \qquad \text{(Eq. 8)}$$

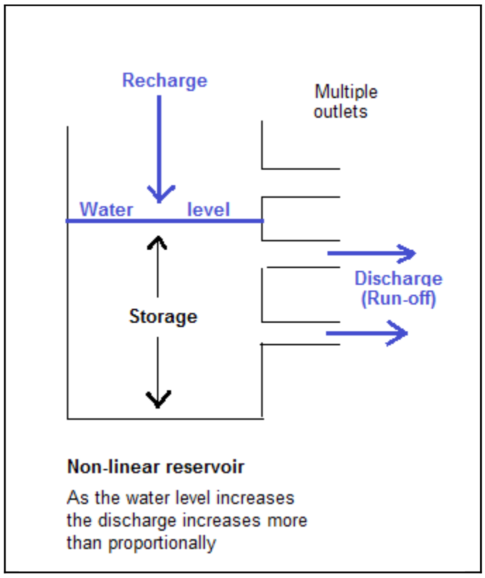


Fig. 3. A non-linear reservoir with multiple outlets whereby the discharge increases more than proportionally with the storage.

The reaction factor can now be written as

$$\alpha = B.Q + C \qquad \text{(Eq. 9)}$$

The RainOffT software [Ref. 4] solves equation 8 numerically and optimizes the values of A and B so that a maximum fit is obtained of the measured Q values to the calculated ones according to the model.

The program also permits to go a step further using a reservoir consisting of two parts (figure. 4)

The original RainOff software [Ref. 3] solves the model described in figure 4 using the equivalent of equation 9:

$$\alpha = \alpha_1 = B_1.Q + C_1 \quad [Q<Q_Z] \qquad \text{(Eq. 9a)}$$
$$\alpha = \alpha_2 = B_2.Q + C_2 \quad [Q>Q_Z] \qquad \text{(Eq. 9b)}$$

where Q_Z is the runoff divide, i.e. the runoff when the lower part of the reservoir is just full and the upper part is empty.

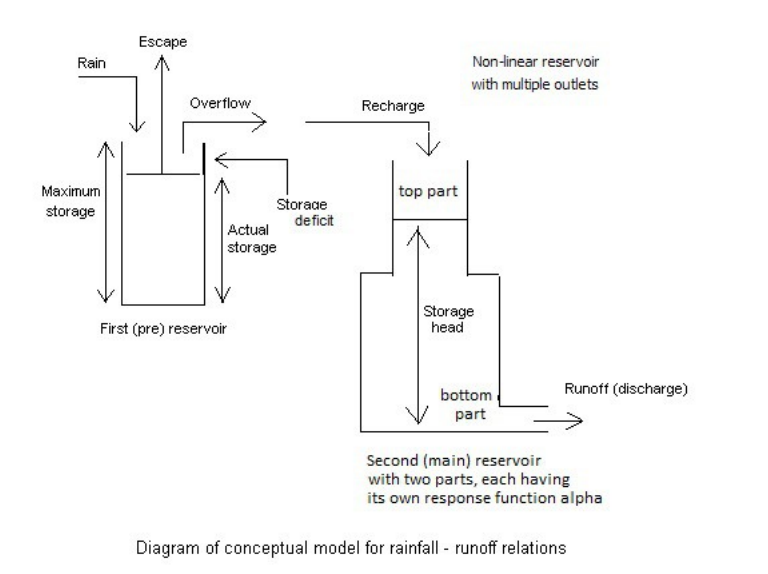


Fig. 4. A reservoir model with two parts. In the top part the storage level, given a unit of rainfall, increases more rapidly than in the lower part.

The second (main) reservoir is preceded by a pre-reservoir that provides the net recharge after deducting evaporation (escape) and replenishment of the soil moisture.

Figure 5 gives an example of the α_1 and α_2 (reservoir functions) obtained from regressions of calculated F values according to equation 9a and b during dry spells on discharge. The separation point here is $Q_Z = 1.15$ mm/hr.

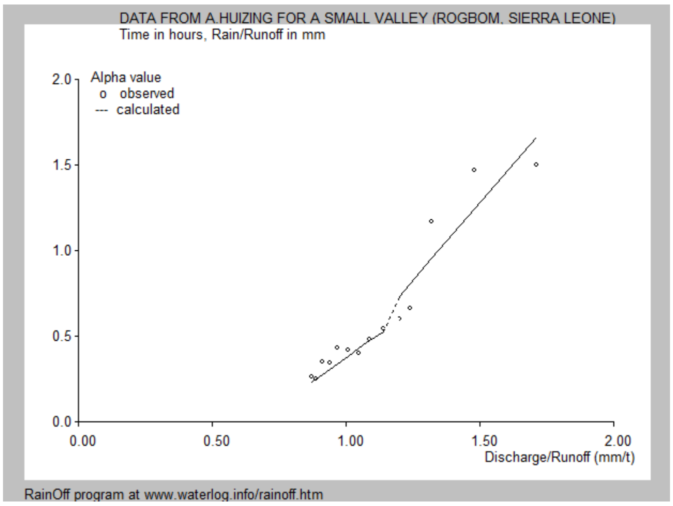


Fig. 5. Example of two reservoir functions below and above a separation point.

When the main reservoir consists of a container with, towards the top, linearly inward sloping walls (figure 6), then one obtains a reservoir reaction factor (F) that is a quadratic function of Q:

$$\alpha = A.Q^2 + B.Q + C \qquad \text{(Eq. 10)}$$

so that equation 9 now becomes:

$$Q = Q_1 \exp\{-(A.Q_1^2 + B.Q_1 + C)(T_2 - T_1)\} + R\,[1 - \exp\{-(A.Q_1^2 + B.Q_1 + C)(T_2 - T_1)\}]$$

(Eq. 11)

The software program RainOffQ [Ref. 3] solves equation 11 numerically and optimizes the values of A, B and C so that a maximum fit is obtained of measured Q values with calculated ones according to the model.

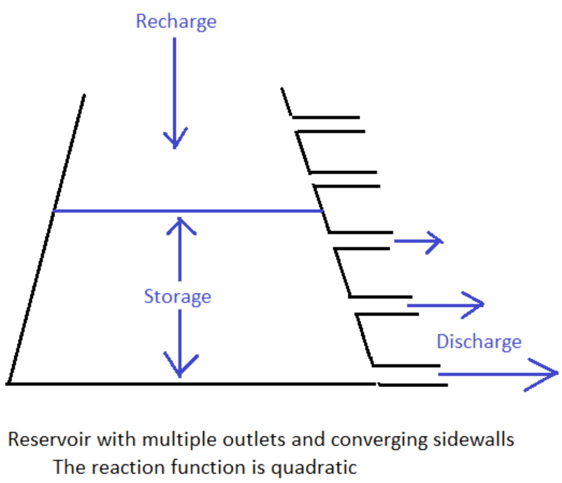


Fig. 6. The main reservoir has converging sides and uses Eq. 10.

2 Application to a valley in Sierra Leone

Gunneweg et al. [Ref. 5] give description of the hydrologic situation and water management systems of small valleys in W. Africa. Figure 7 sketches some of these characteristics and figure 8 gives a picture of a small valley with rice cultivation.

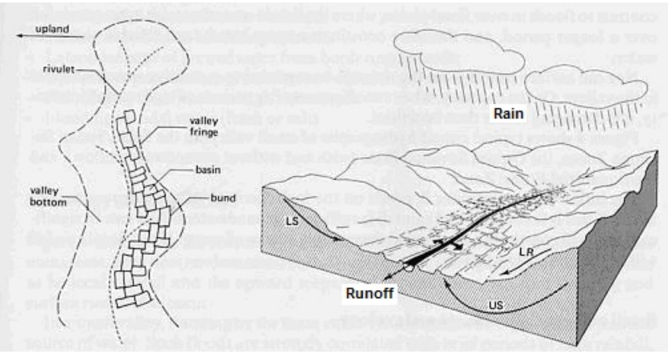


Fig. 7. Sketch of physical and hydrological characteristics of a small valley with rice fields in Sierra Leone [Ref. 5]

Fig. 8. A small valley with rice cultivation in W. Africa. The central drain is temporarily overgrown. [Ref. 5]

Huizing [Ref. 6] collected hourly rainfall-runoff relations in a small cultivated valley (Rogbom) in Sierra Leone near the township Makenni, during the months of July and August 1987. Measurements were made on 6 days spaced apart and two continuous periods of 10 days.

Of the 6 separate days there were 5 with considerable rainfall in the morning followed by a dry afternoon.

These days are July 13 and 20, August 6 and 24. and 17 September.

Figure 8 shows the analysis, using the RainOffT version, for August 6, which day was selected because there were two rainy spells, while the other days only had only one. The coefficient of explanation (96%) is quite high. The reaction factor is found as $\alpha = 0.185\ Q\ - 0.176$.

Figure 10 confirms that the runoff increases more quickly at higher rainfalls so that a non-linear reservoir is appropriate.

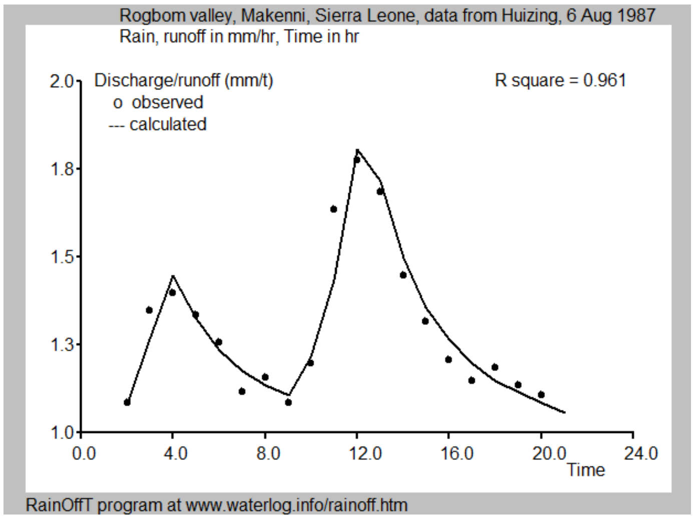


Fig. 9. RainOff results: calculated and observed hourly runoff, Rogbom valley, August 6, 1987.

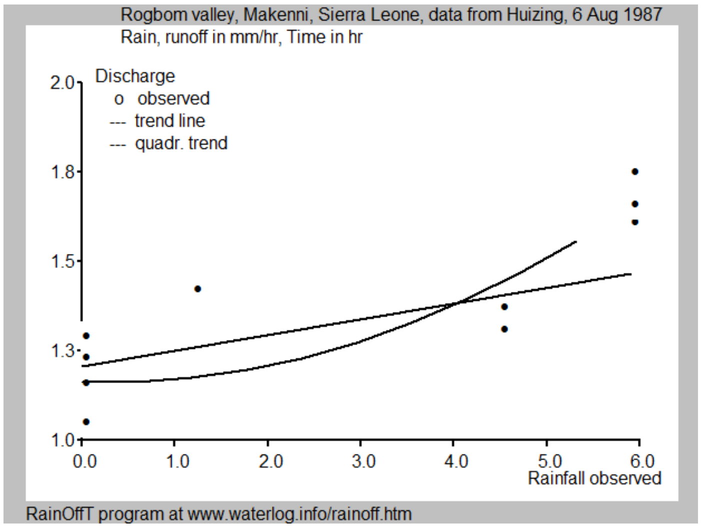


Fig. 10. RainOff results: observed hourly rainfall and runoff, Rogbom valley, August 6, 1987.

The quadratic up-curving trend in figure 10 suggests that the runoff increases more rapidly at higher rainfalls as is expected by the RainOff non-linear reservoir model

Figure 11 depicts the runoff simulation for a 10-day period (17-27 August 1987).

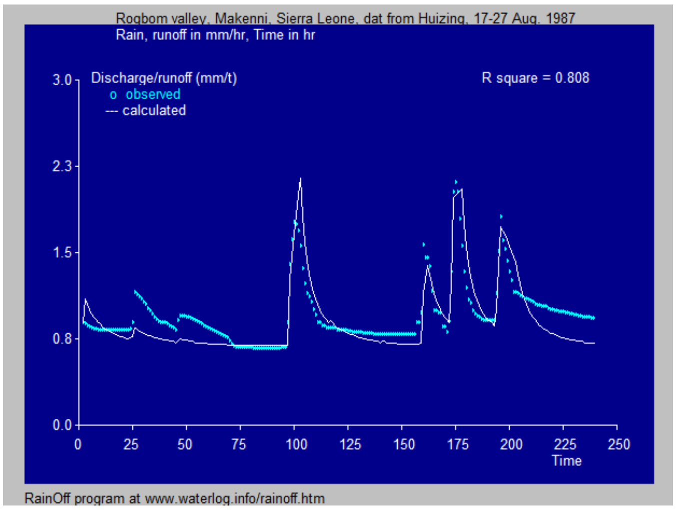


Fig. 11. RainOff results: calculated and observed hourly runoff, Rogbom valley, period 17–27 August 1987.

In Figure 11 the fit of the model to the data is not perfect, the coefficient of determination is relatively low (0.81 or 81%). For this there are two reasons:

1 – There are many days (107) with constant runoff during dry spells , while the model expects the runoff to decrease gradually during such periods. In the valley bottom rice is cultivated, which makes that the runoff is influenced by farmers and that the runoff conditions are not entirely natural. RainOff, on the other hand, assumes natural conditions without interference by mankind during the runoff process.

2 – The trend of observed runoff versus rainfall is one of a gradually smaller runoff increase with increasing rainfalls (figure 12). This is the opposite of the trend shown in figure 8, and it is not in accordance with the assumptions made for the non-linear model.

For both these reasons, the RunOff software is not able to handle the rainfall-runoff relation over a longer period adequately. These two adverse features are also not in accordance to the generally accepted hydrological assumptions.

From figure 11, it appears that day 21 August (starting at 96 hrs) and days 24 and 25 August (from hour 168 to hour 188) would be suitable to apply the model.

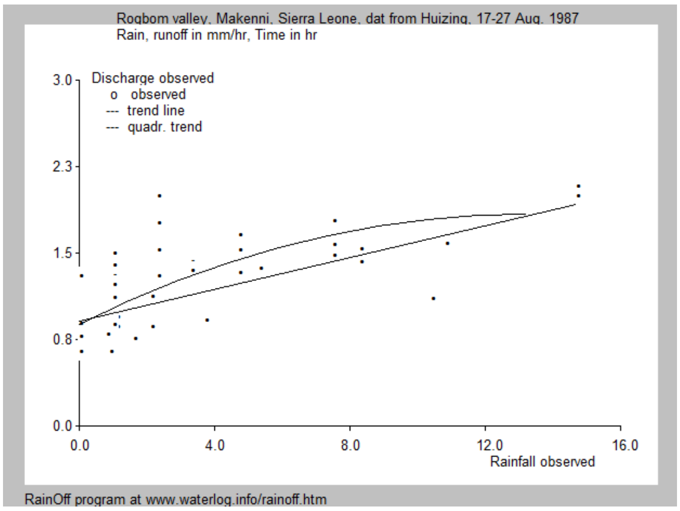


Fig. 12. RainOff results: observed hourly rainfall and runoff, Rogbom valley, period 17–27 August 1987.

On this basis, the period of 23-24 August was selected for closer inspection because there are two runoff peaks and there is no period with constant runoff during several days.

The excellent data fit to the runoff model for this period is shown in figure. 13. The coefficient of determination is quite high (96%).

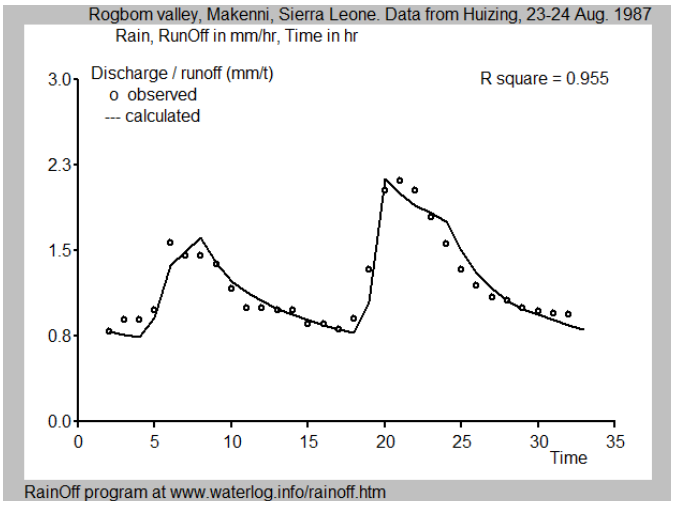


Fig. 13. RainOff results: observed hourly rainfall and runoff, Rogbom valley, period 23 – 24 August 1987.

3 Verification

For verification it is required to use the parameters of the reservoir model obtained during the calibration rounds and apply these to a data set that was not analyzed before. However, the parameters for the examples given before produced large differences amongst the different examples (table 1). Apparently, the environmental

Table 1. Parameters of the reservoir reaction factor (α = B.Q + C) for the RainOff model employing a reservoir consisting of two parts (Fig. 3):

Date of data	Runoff separation point (Q_Z, mm/hr)	Runoff (Q) Condition	B coefficient	C value
6 August	1.07	$Q > Q_Z$	0.1887	− 0.1525
		$Q < Q_Z$	0.0000	0.0409
23-24 August	0.98	$Q > Q_Z$	0.1145	− 0.0266
		$Q < Q_Z$	0.1305	− 0.0366
17 – 27 August	1.31	$Q > Q_Z$	0.0000	0.0584
		$Q < Q_Z$	0.1193	− 0.0771

Apparently, the environmental conditions change strongly from time to time, which prevents the use of a standard set of parameters for all the months of the summer period studied (July, August and September). Possibly, one reason for this variation is the vegetative development of the rice crop cultivated in the bottom of the valley, together with heightening and strengthening of the bunds around the rice field when the crop grows bigger, as well as diverting runoff water for irrigation Yet, verification can be done within the month.

In figure 14, a screen shot of the input tab-sheet of the RainOff program, the parameters A_1, B_1, A_2 and B_2 of the reaction factors ($\alpha_1 = B_1.Q+C_1$ and $\alpha_2 = B_2.Q + C_2$) and the separation point Q_Z determined for the period 17 – 27 August (Fig. 11, Table 1), were used to simulate the runoff from the rainfall data for the period 22/34 August (Fig.15).

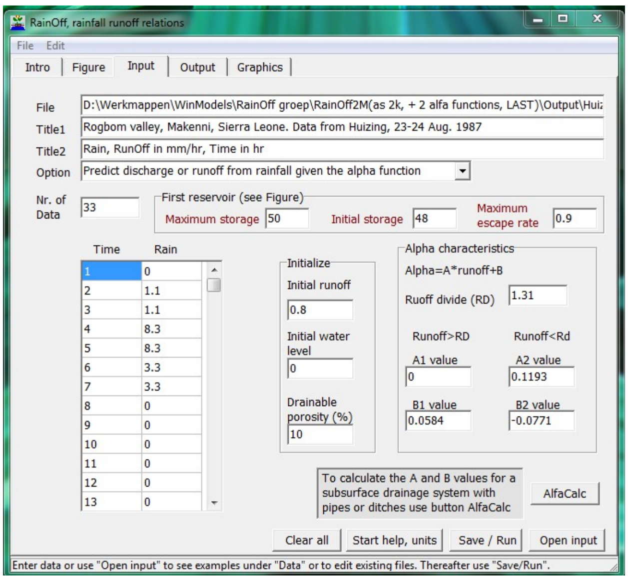


Fig. 14. Screen-print of input tab-sheet of the RainOff program showing the data for runoff simulation of period 23-24 August

In figure 15 the characteristics of the reservoir function α (in the figure called Alpha) derived from the period 17-27 August as shown in table 1 have been entered.

The results of the simulation are shown in figure 15 and compared with the measured runoff.

The correspondence is reasonable, taking into account the environmental changes that may have occurred.

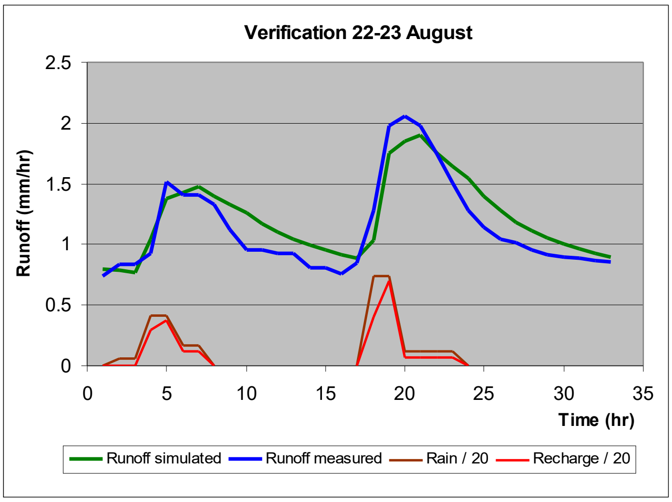


Figure 15. The runoff hydrograph for the period of 22-23 August (blue line, see also Fig. 13) was simulated (green line) on the basis of rainfall using the parameters of the reservoir function α derived for the period 17–27 August (see Fig. 11 and Table 1).

4 Conclusion

The RainOff model has produced reliable results in short term (1 or 2 day) simulations (figures 8 and 12). This leads to the conclusion that the software is valuable. The parameters of the reservoir functions, however, were quite different from month to month, probably due to changing environmental conditions and rice cultivation practices in the valley. Verification between months, therefore, is not possible. However, verification within the month of August has produced an acceptable, though not perfect, agreement of simulated and measured runoffs.

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