STANDARD ERROR OF THE BREAKPOINT FOR TYPE 2 IN SEGREG
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Used in the SegReg program at https://www.waterlog.info/segreg.htm for segmented regression
On website https://www.waterlog.info public domain, latest upload 20-11-2017

The regression line to the left of the breakpoint (BP) is represented as

$$y = a_1 (x - \chi_1) + \eta_1$$

and to the right as

$$y = a_2 (x - \chi_2) + \eta_2$$

where y is the dependent variable, x the independent variable, a_1 the regression coefficient of the line to the left of BP, a_2 the regression coefficient to the right, χ_1 the average of the x values to the left of BP, χ_2 the average of the x values to the right , η_1 the average of the y values to the left of BP and η_2 the average of the y values to the right.

At the breakpoint, where x=BP, the y values are equal, so that here

$$a_1 (bp - \chi_1) + \eta_1 = a_2 (bp - \chi_2) + \eta_2$$

where bp is the value of x at BP.

Re-arrangement gives

$$bp = (a_1\chi_1 - \eta_1 - a_2\chi_2 + \eta_2) / (a_1 - a_2)$$

The standard error S of BP can be found, using the laws of propagation of errors in additions and multiplications, from

$$S^2_{bp} = S^2_{\{1/(a1 - a2)\}} (a_1\chi_1 - \eta_1 - a_2\chi_2 + \eta_2)^2 + \{1/(a_1 - a_2)^2\} S^2_{\{a1\chi1 - \eta1 - a2\chi2 + \eta2\}}$$

where

$$S^2_{\{1/(a1 - a2)\}} = S^2_{(a1 - a2)} / (a_1 - a_2)^4$$

with

$$S^2_{(a1 - a2)} = S^2_{a1} + S^2_{a2}$$

and

$$S^2_{\{a1\chi1 - \eta1 - a2\chi2 + \eta2\}} = S^2_{a1\chi1} + S^2_{\eta1} + S^2_{a2\chi2} + S^2_{\eta2}$$

with

$$S^2_{a1\chi1} = a^2_1 S^2_{\chi1} + \chi_1{}^2 S^2_{a1}$$
$$S^2_{a2\chi2} = a^2_2 S^2_{\chi2} + \chi_2{}^2 S^2_{a2}$$

For the determination of the standard deveiation of a_1 , a_2 , χ_1 , χ_2 , η_1 and η_2 see for example http://www.waterlog.info/pdf/regtxt.pdf