Influence of the soil's drainable porosity on the functioning of a subsurface drainage system analyzed with a model for drainage calculations.

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Abstract

The functioning of a subsurface drainage system depends, among other factors, on the drainable porosity of the soil. However, for the determination of the drainable porosity one needs a large number of laborious pF curves that may, moreover, reveal a large variation. The use of a computer program like DrainCalc may be instrumental to overcome this problem. This software uses the principles of a non-linear reservoir model with a reaction (response) function consisting of two components, one for flow above and one for flow below drain level including the determination of the equivalent depth. The reaction (response) function is calculated on the basis of the parameters (characteristics, properties) of the drainage system, including the drainable porosity, and use is made of existing non-steady state drainage equations. In this paper, the principles of DrainCalc and the functioning of its user interface are explained. Further, the influence of the soil's drainable porosity will be illustrated with examples of subsurface drainage systems working under rain-fed conditions and in irrigated lands without rainfall.

Contents

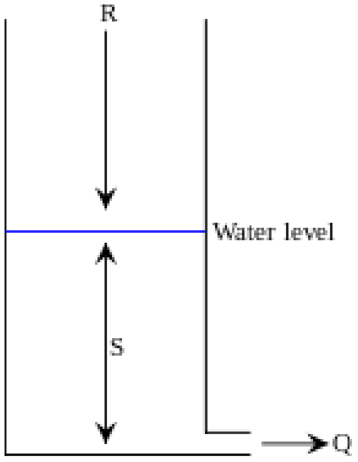
1. Introduction

The free DrainCalc program [*Ref. 1*] is based on the RainOffT program [*Ref. 2*], but it is more directly dealing with subsurface drainage systems and it has the additional facility to determine the reaction (response) function from data on the level of the groundwater table above the drains. The RainOffT model has initially been used to analyze the rainfall – runoff relations of a small valley in Sierra Leone [*Ref. 3*] and in Germany [*Ref. 4*]. Later, it was used to analyze agricultural subsurface drainage systems [*Ref. 5, Ref. 6*].

2. Principles of the DrainCalc model

2.1 Linear and non-linear reservoir, discharge equations

DrainCalc is built on the principles of a non-linear reservoir, an extension of the linear reservoir. The linear reservoir was described by D.A.Kraijenhof van de Leur [*Ref. 8*] and its principles are given in *figure 1*.

	R = recharge {dimension L/T} S = storage {dimension L} Q = discharge, runoff {dimension L/T}	***Figure. 1****.* *The concept of a linear reservoir.*

The reservoir (reaction, response) function is:

$Q = \alpha . S$ (Eq. 1)

where α = a constant reaction factor {1/T}

Differentiating S to time T gives

$dS/dT = d(Q/\alpha)/dT = R-Q$ (Eq. 2)

Integrating Eq. 2 with limits Q_1, Q_2, T_1 and T_2 yields:

$$Q_2 = Q_1 \exp\{-\alpha (T_2 - T_1)\} + R\,[1 - \exp\{-\alpha (T_2 - T_1)\}] \qquad \text{(Eq. 3)}$$

where Q_2 and Q_1 are Q at time T_2 and T_1 respectively.

With Equation 3 the discharge Q_2 can be calculated from R, Q_1, α, and the time difference.

This concept is often to simple to characterize the watershed as its reaction factor is usually more complicated. Therefore Nash [Ref. 8] employed a cascade of linear reservoirs, one reservoir emptying into the next, while Kraijenhoff [Ref. 9] used a number of parallel reservoirs over which the rainfall is distributed in some proportion, while the reservoirs joined their discharge.

In hydrology, the concept of non-linear reservoirs has seldom been applied. Instead of a reservoir with a constant reaction factor, one could employ a non-linear reservoir with a reaction factor that changes with storage (*figure 2*) instead of being a constant, thus avoiding the difficulty of dealing with a series of reservoirs.

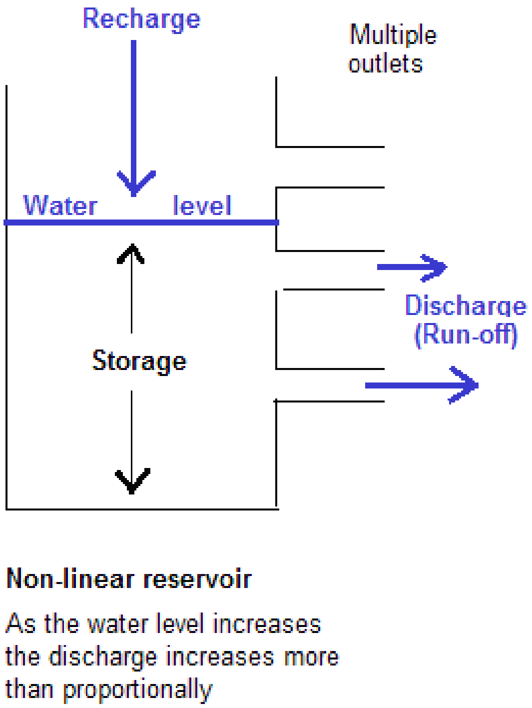


Figure 2*.*

A non-linear reservoir with multiple outlets whereby the discharge increases more than proportionally with the storage.

The equivalents of equation 1, 2 and 3 for the non-linear reservoir are equations 4, 5 and 6 as follows [*Ref. 9*]:

$$Q = (A.Q + C).S \qquad \text{(Eq. 4)}$$
$$dS/dt = R-(A.Q+C).S = R - A.Q.S + C.S \qquad \text{(Eq. 5)}$$
$$Q_2 = Q_1 \exp \{ -(A.Q_1+C).(T_2-T_1) \} + R[1-\exp\{-(A.Q_1+C).(T_2-T_1)\} \qquad \text{(Eq. 6)}$$

The reaction function (reservoir or response function) can now be written as

$$\alpha = A.Q + C \qquad \text{(Eq. 7)}$$

It is no longer a constant, but it depends on the discharge. The factor B and the term C are found by DrainCalc with a numerical (calibration) method, varying the B and C values and selecting the combination that maximizes the fit of the simulated discharge/runoff in time to the observed one.

The values B and C represent the properties (characteristics) of the drainage system.

It is also possible to use a quadratic α function: $\alpha = A.Q^2 + B.Q + C$ [Ref. 2]. The software for this case is called RainOffQ. In some, but not all, cases it gives a still better result.

2.2 Recharge of the groundwater table

The recharge depends on the rainfall or irrigation and the escape factors like evaporation and percolation to an aquifer with natural drainage. When the percolation is taken negative it will represent upward seepage from the aquifer. The rainfall enters a pre-reservoir with a storage function as shown in *figure 3*.

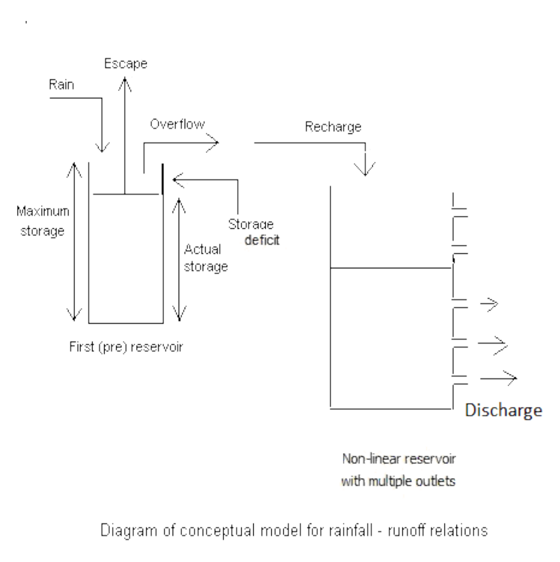


Figure 3.

The second (main) reservoir is preceded by a pre-reservoir that provides the net recharge of the groundwater table after deducting evaporation (escape) and replenishment of the soil moisture

The “escape” usually consists of evaporation, but it may include percolation to the aquifer and natural drainage, while upward seepage from the aquifer can be considered as a negative Escape. The Recharge is thus found from:

Recharge = Overflow = Rain – Escape – Storage Deficit. (Eq. 8)

During rainy periods the Storage Deficit can become zero and the Recharge of the groundwater table will equal the Rainfall less Evaporation. In dry periods the Escape may exceed the Rainfall and the Storage Deficit will then increase.

2.3 Hydraulic head (level of the groundwater table) equation

For subsurface drainage systems the hydraulic head H (height of the groundwater table above drain level) can be calculated in a way similar to equation 6 [Ref 7]:

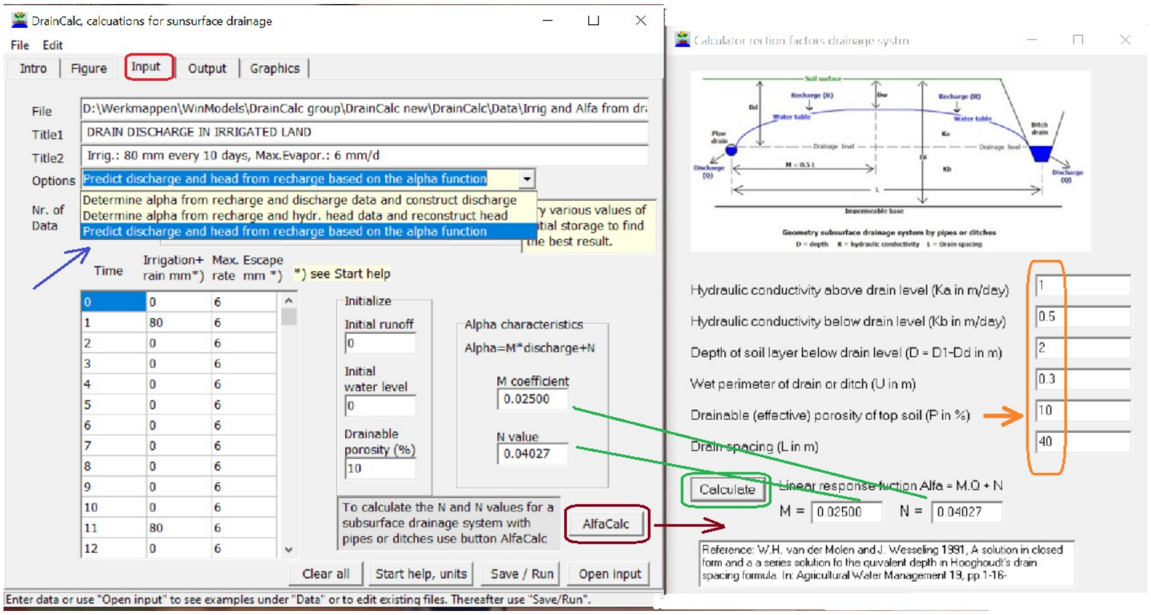
$$H_2 = H_1 \exp(-\alpha.Tt) + R[1-\exp(-\alpha.Tt)/0.8Pd.\alpha] \qquad \text{(Eq. 9)}$$

where $Tt = T_2 - T_1$ and *Pd* is the drainable porosity of the soil (m^3 of pores per m^3 of soil). It equals the soil moisture content at saturation less the content at field capacity where the moisture is retained after being drained.

3. User interface of the DrainCalc program

Reservoir response function determined from drainage system properties (parameters)

Figure 4 depicts the user interface of the input menu (red square) for the determination of the reservoir (reaction, response) function Alpha = B*discharge + C from the drainage properties (parameters, characteristics, blue arrow)



File D:\Werkmappen\WinModels\DrainCalc group\DrainCalc new\DrainCalc\Data\Irrig and Alfa from dr

Title1 DRAIN DISCHARGE IN IRRIGATED LAND

Title2 Irrig.: 80 mm every 10 days, Max.Evapor.: 6 mm/d

Options Predict discharge and head from recharge based on the alpha function

Determine alpha from recharge and discharge data and construct discharge
Determine alpha from recharge and hydr. head data and reconstruct head
Predict discharge and head from recharge based on the alpha function

Nr. of Data

y various values of
itial storage to find
the best result.

*) see Start help

Time	Irrigation+ rain mm*)	Max. Escape rate mm *)
0	0	6
1	80	6
2	0	6
3	0	6
4	0	6
5	0	6
6	0	6
7	0	6
8	0	6
9	0	6
10	0	6
11	80	6
12	0	6

Initialize
Initial runoff: 0
Initial water level: 0
Drainable porosity (%): 10

Alpha characteristics
Alpha=M*discharge+N
M coefficient: 0.02500
N value: 0.04027

To calculate the N and N values for a subsurface drainage system with pipes or ditches use button AlfaCalc

Enter data or use "Open input" to see examples under "Data" or to edit existing files. Thereafter use "Save/Run".

Hydraulic conductivity above drain level (Ka in m/day): 1
Hydraulic conductivity below drain level (Kb in m/day): 0.5
Depth of soil layer below drain level (D = D1-Dd in m): 2
Wet perimeter of drain or ditch (U in m): 0.3
Drainable (effective) porosity of top soil (P in %): 10
Drain spacing (L in m): 40

Linear response fuction Alfa = M.Q + N
M = 0.02500 N = 0.04027

Reference: W.H. van der Molen and J. Wesseling 1991, A solution in closed form and a a series solution fo the quivalent depth in Hooghoudt's drain spacing formula. In: Agricultural Water Management 19, pp.1-16-

***Figure 4**. User interface of the input menu (red square) for the determination of the reservoir function Alpha = B*discharge + C from the drainage properties (parameters, characteristics, blue arrow). When clicking on the "AlfaCalc" button (purple square) on finds the calculator for the components B and C of the Alpha function to the right (purple arrow). After completing the characteristics (parameters, properties) of the subsurface drainage system (orange square) as illustrated in the figure (at the right-hand top) and clicking on the "Calculate" button (green square) the B and C values are computed and entered in the respective edit blocks (green lines). The input of the drainable porosity is indicated by an orange arrow.*

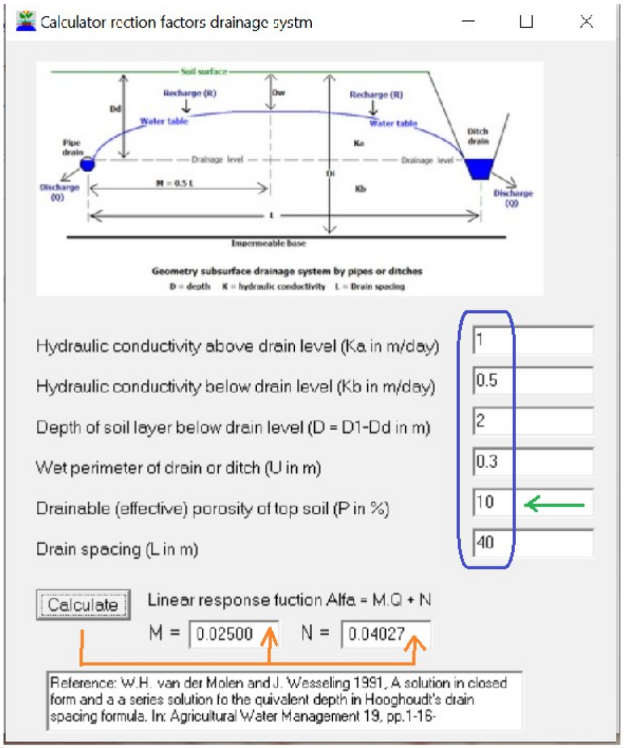
The other two application options shown in *figure 4* above in yellow color between the blue lines are not used in this paper. However, the choice of the first of these two options has been used in the following two publications:

A - The free RainOffT model, useful for analyzing the hydrology of subsurface drainage systems in transient (non-steady) state, Section 3.2. On line:
https://www.waterlog.info/pdf/RainOff applications.pdf
or:
https://www.researchgate.net/publication/357867025_The_free_RainOffT_model_useful_for_analyzing_the_hydrology_of_subsurface_drainage_systems_in_transient_non-steady_state

B - RainOff, a rainfall-runoff model applied to a subsurface drainage system by calibration and validation, Section 4.. On line:
https://www.waterlog.info/pdf/Drainage model.pdf
or:
https://www.researchgate.net/publication/349836058_RainOff_a_rainfall-runoff_model_applied_to_a_subsurface_drainage_system_by_calibration_and_validation

Also examples can be seen in the data file accompanying the DrainCalc program once downloaded [*Ref. 1*].

For more clarity, an enlargement of the illustration at the right-hand top of *figure 4* is presented in *figure 5:*



Hydraulic conductivity above drain level (Ka in m/day) 1

Hydraulic conductivity below drain level (Kb in m/day) 0.5

Depth of soil layer below drain level (D = D1-Dd in m) 2

Wet perimeter of drain or ditch (U in m) 0.3

Drainable (effective) porosity of top soil (P in %) 10

Drain spacing (L in m) 40

Linear response fuction Alfa = M.Q + N

M = 0.02500 N = 0.04027

Reference: W.H. van der Molen and J. Wesseling 1991, A solution in closed form and a a series solution to the quivalent depth in Hooghoudt's drain spacing formula. In: Agricultural Water Management 19, pp.1-16-

Figure 5.

Enlargement of the illustration at the right-hand top of figure 4.

The orange square is replaced by a blue square indicating the drainage properties to be given, while the orange arrow is now represented by a green one to stress the place where the drainable porosity has to be entered.

"Clicking on the "calculate button produces the components M and N of the Alpha function, see the orange arrows.

4. Examples with data from literature

The effects of the soil's drainable porosity on the subsurface drainage functioning are investigated for the drain discharge and the hydraulic head (height of the groundwater table above drain level). To clarify this concept the illustration at the top right of figure 4 is enlarged hereunder (*figure 5*).

4.1 Subsurface drainage under rain-fed conditions without irrigation

The functioning of the subsurface drainage system in dependence of the rainfall and DIL is shown in the following examples. The parameters of the subsurface drainage system in the model can be found in the *.par files produced by the program. In these examples they are as follows:

***Table 1**. Parameters of the subsurface drainage system in the examples.*

Hydraulic conductivity above drain level (Ka, m/day	0.5
Hydraulic conductivity below drain level (Kb, m/day)	1.0
Depth of soil layer below drain level (D1-Dd, m)	2.0
Wet perimeter if drain or ditch (U, m)	0.3
Drainable (effective) porosity of top soil (*Pd*, %)	3, 5, 7.5 and 10% respectively
Drain spacing (L, m)	30

The calculation of the reservoir reaction (response) functions in dependence of the drainable porosity Pd, referring to *equation 7,* is given in the next table.

***Table 2**. The reservoir reaction (response) function as influenced by* Pd

Pd = 3%	α = 0.0741*Discharge + 0.4483
Pd = 5%	α = 0.0444*Discharge + 0.2690
Pd = 75%	α = 0.0296*Discharge + 0.1793
Pd = 10%	α = 0.0222*Discharge + 0.1345

The rainfall and resulting recharge of the groundwater table in this example are depicted in *Figure 6*.

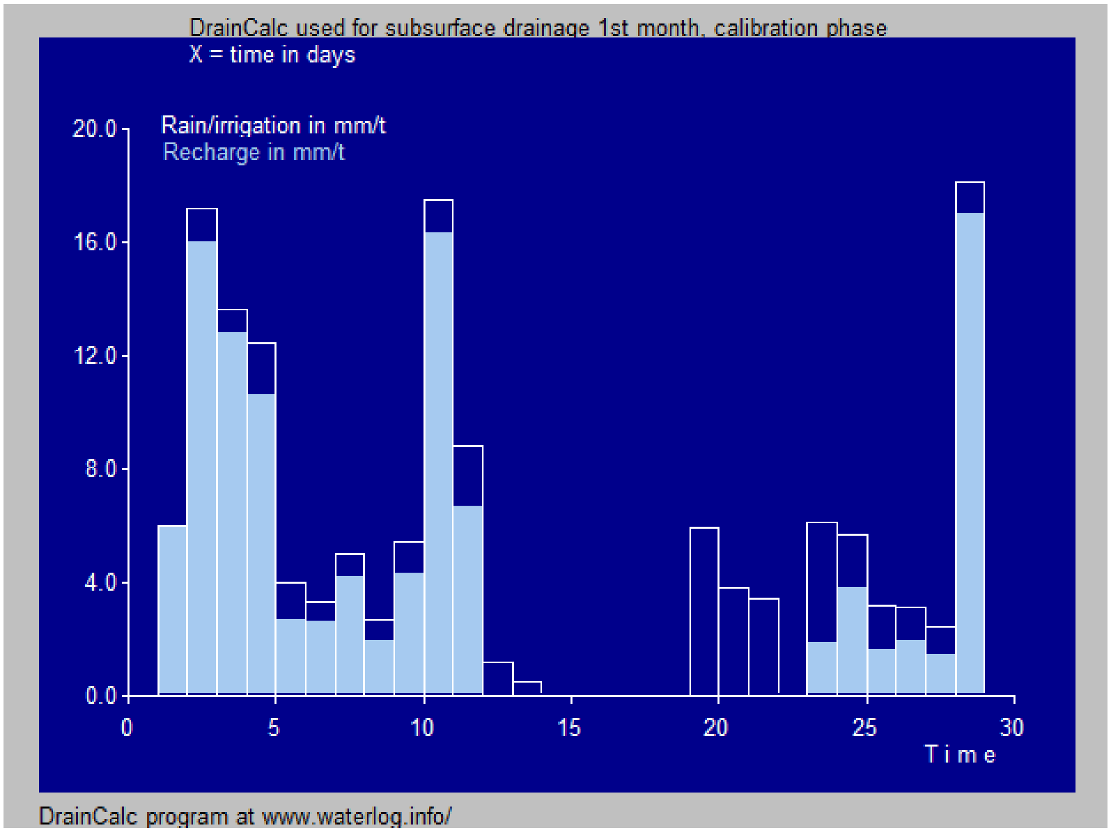


Figure 6. *Rainfall and recharge of the groundwater table. Not all rainfall leads to recharge because part of the rainfall is evaporated and during longer periods between the rainfalls the evaporation creates a soil moisture deficit that is replenished with the next rainfall before recharge can occur (*equation 8*).*

The following *figures 7 and 8* reveal the behavior of the hydraulic head and the drain discharge under varying drainable porosity (*Pd*) conditions:

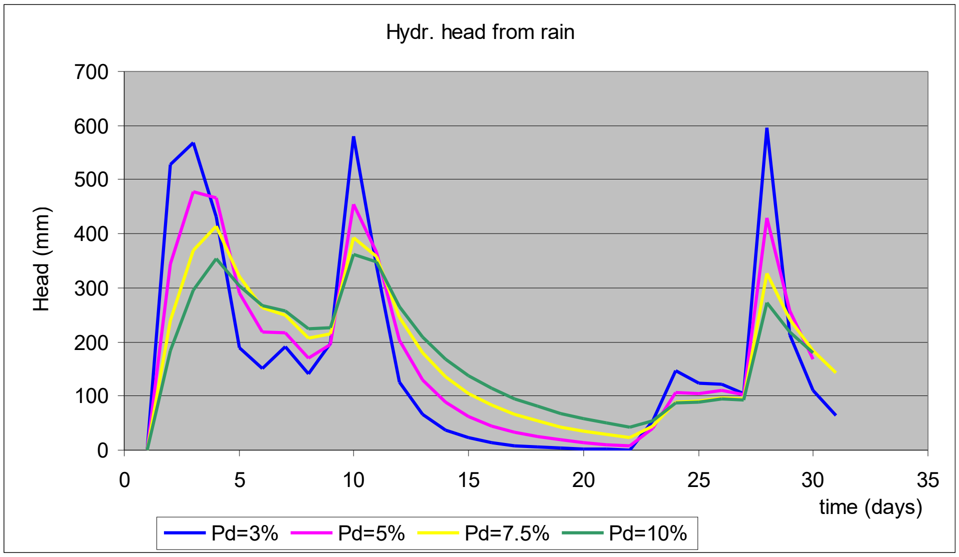


Figure7.

*The hydraulic head (H) caused by recharge of the groundwater table resulting from the rainfall (*equation 9*) as influenced by* Pd.

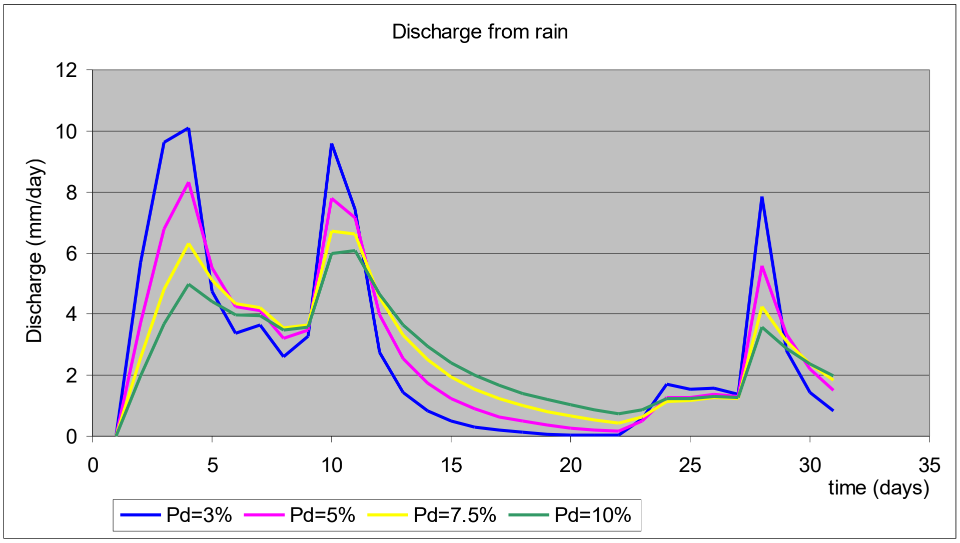


Figure 8.

*The drain discharge rate (Q) caused by recharge of the groundwater table resulting from the rainfall (*equation 6*) as influenced by* Pd.

Figure 7 demonstrates that the maximum head reduces considerably when the *Pd* changes 3% (blue line) to 7.5% (yellow line). On the other hand, when the *Pd* changes from 7.5% to 10% (green line), the reduction is small. So, when the *Pd* is greater than 10% it is hardly influential. For the minimum head the reverse is true.

According to *figure 8*, the peak discharge is highest at the lowest *Pd* value (3%), see the blue line, and lowest for the highest *Pd* (10%, green line). However, when the discharge reduces after a period without rainfall, the lower discharge rates reveal the opposite trend.

4.2 Subsurface drainage under irrigation conditions without rainfall

The functioning of the subsurface drainage system in dependence of irrigation and DIL is shown in the following examples. The parameters of the subsurface drainage system in the model can be found in the *.par files produced by the program. In these examples they are as follows:

***Table3**. Parameters of the subsurface drainage system in the examples*.

Hydraulic conductivity above drain level (Ka, m/day	0.5
Hydraulic conductivity below drain level (Kb, m/day)	1.0
Depth of soil layer below drain level (D1-Dd, m)	2.0
Wet perimeter if drain or ditch (U, m)	0.3
Drainable (effective) porosity of top soil (*Pd*, %)	3, 5, 7.5, and 10% respectively
Drain spacing (L, m)	40

These data are the same as in *Table 1* for the rainfall case, except that the drain spacing here is 40 m instead of 30 m.

The calculation of the reservoir reaction (response) functions in dependence of *Pd*, referring to *equation 7* is given in the next table.

***Table 4**. The reservoir reaction (response) function) as influenced by* Pd

Pd = 3%	$\alpha = 0.0833*\text{Discharge} + 0.1342$
Pd = 5%	$\alpha = 0.0500*\text{Discharge} + 0.0806$
Pd = 75%	$\alpha = 0.0333*\text{Discharge} + 0.0537$
Pd = 10%	$\alpha = 0.0250*\text{Discharge} + 0.0660$

The irrigation and recharge of the groundwater table in this example are depicted in *Figure 9*.

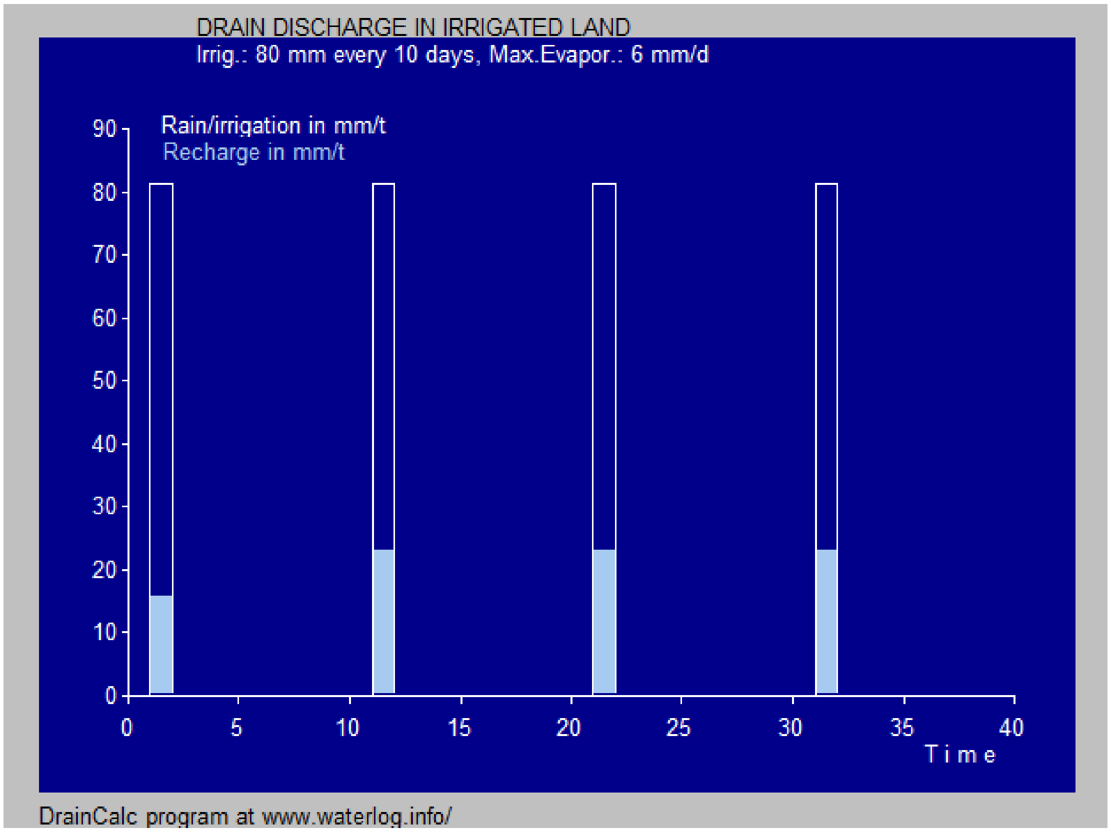


Figure 9. *Irrigation and recharge of the groundwater table. The irrigations are given every 10 days and amount 80 mm each. Not all irrigation water leads to recharge because a large part of the irrigation is evaporated by the crops and during periods between the irrigations the evaporation creates a soil moisture deficit that is replenished with the next irrigation before recharge can occur (*equation 8*).The recharge is also call irrigation loss or excess, but it is required to maintain a proper salt balance in the root zone of the plants* [Reference 11].

The following *figures 10 and 11* reveal the behavior of the hydraulic head and the drain discharge under varying *Pd* conditions

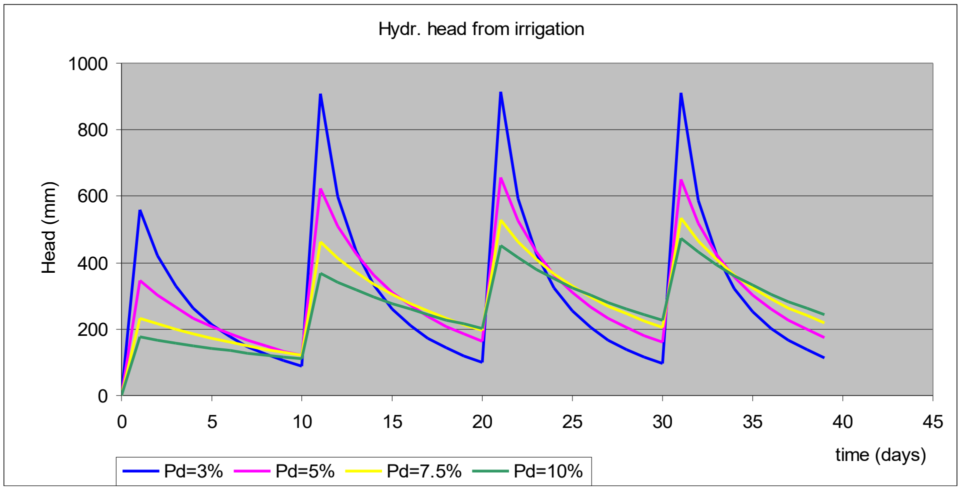


Figure10.

The hydraulic head (H) caused by recharge of the groundwater table resulting from the irrigation (equation 9) as influenced by Pd.

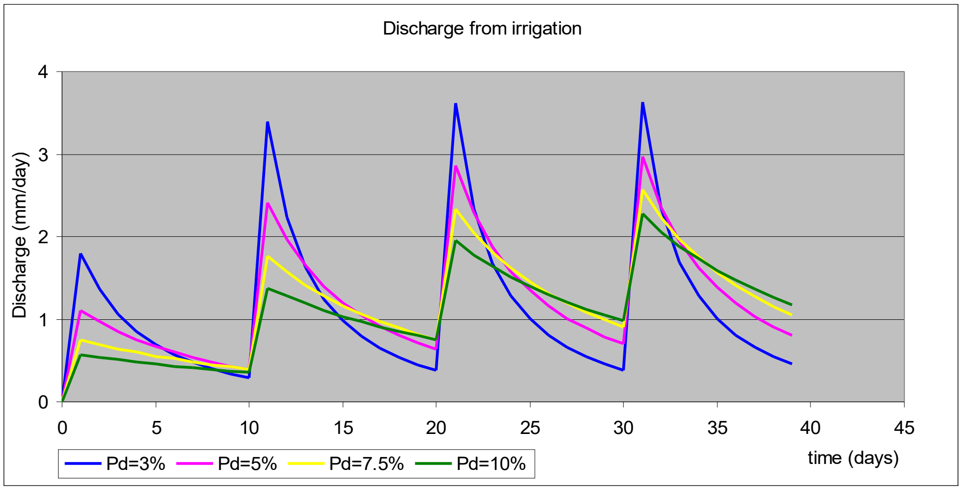


Figure 11.

The drain discharge rate (Q) caused by recharge of the groundwater table resulting from the irrigation (equation 6) as influenced by Pd.

Figure 10 demonstrates that the head reduces considerably when the *Pd* changes from 3% (blue line) to 7.5% (yellow line). On the other hand, when the *Pd* changes from 7.5% to 10% (green line) the reduction is relatively small. So, when the *Pd* value is greater than 10% it is hardly influential.

According to *figure 11*, the peak discharge is highest at the lowest *Pd* value (3%), see the blue line, and lowest for *Pd* = 10% (green line). However, when the discharge reduces after a period without irrigation, the lower discharge rates for *Pd* > 3% come close together.

5. Conclusion

When the spacing of subsurface drains has to be determined for an area needing the installation of a subsurface drainage system, it is advisable to assume different depths of the impermeable layer below drain level (DIL) in the calculations. When, from the soil profile, it can be decided that the DIL value is greater than 10 to 20% of the drain spacing, a precise investigation of the DIL value is not required. Otherwise it may be effective to perform an additional research on this matter.

6. References

[Ref. 1] DrainCalc, free model software for analysis of the level of the groundwater table and drain discharge in subsurface drainage systems:
https://www.waterlog.info/draincalc.htm

[Ref. 2] RainOff, RainOffT and RainOffQ, model software for rainfall-runoff analysis:
https://www.waterlog.info/rainoff.htm

[Ref. 3] R.J. Oosterbaan, 2019. *Rainfall-runoff relations of a small valley in Sierra Leone with a non-linear reservoir model. International Journal of Environmental Science, **4**, 1-9 , 2019.*
On line: https://www.waterlog.info/pdf/SLeone.pdf
or*:*
https://www.researchgate.net/publication/332466264_RAINFALL-RUNOFF_RELATIONS_OF_A_SMALL_VALLEY_ASSESSED_WITH_A_NON-LINEAR_RESERVOIR_MODEL

[Ref. 4] Rainfall and runoff data of the "Herbornseelbach" catchment (watershed), Hesse, Germany, evaluated with the RainOff model by calibration and validation of catchment parameters. On line: https://www.waterlog.info/pdf/Hesse Herborn.pdf
or:
https://www.researchgate.net/publication/349662750_Rainfall_and_runoff_data_of_the_Herbornseelbach_catchment_watershed_Hesse_Germany_evaluated_with_the_RainOff_model_by_calibration_and_validation_of_catchment_parameters

[Ref. 5] Simulating subsurface drain discharge and depth of the water table in transient (non-steady) state using the RainOff model. On line:
https://www.researchgate.net/publication/343943124_Simulating_subsurface_drain_discharge_and_depth_of_the_water_table_in_transient_non-steady_state_using_the_RainOff_model

[Ref. 6] The free RainOffT model, useful for analyzing the hydrology of subsurface drainage systems in transient (non-steady) state. On line: https://www.waterlog.info/pdf/RainOff applications.pdf
or:
https://www.researchgate.net/publication/357867025_The_free_RainOffT_model_useful_for_analyzing_the_hydrology_of_subsurface_drainage_systems_in_transient_non-steady_state

[Ref. 7] H.P. Ritzema, 1994. *Subsurface Flow to drains*. Chapter 8 in: H.P. Ritzema (editor), Drainage Principles and Applications, ILRI Publication 16. International Institute for Land Reclamation and Improvement, Wageningen, The Netherlands. On line:
https://edepot.wur.nl/149491
or:
https://www.researchgate.net/publication/272483377_Subsurface_flow_to_drains

[Ref. 8] J.E.Nash, 1958. *Determining runoff from rainfall*. Proc. Inst. Civ. Engs. 10 : p. 163 – 184. On line: https://www.icevirtuallibrary.com/doi/10.1680/iicep.1958.2025
Software: https://www.waterlog.info/nashmod.htm

[Ref. 9] D.A.Kraijenhoff van de Leur, 1958. *A study of non-steady state groundwater flow with special reference to a reservoir coefficient*. De Ingenieur 70: p. 387 – 394. On line:
https://library.wur.nl/WebQuery/hydrotheek/604860

[Ref. 10] Rainfall-runoff model with non-linear reservoir. On line:
https://www.waterlog.info/pdf/reservoir.pdf

[Ref. 11] LeachMod, based on water and salt balances, is a model for the leaching of saline soils and reclamation (improvement, amelioration) of salty areas in irrigated lands by a subsurface drainage system including the simulation of the depth of the water table. On line: https://www.waterlog.info/pdf/leaching model.pdf
or:
https://www.researchgate.net/publication/358229189_LeachMod_based_on_water_and_salt_balances_is_a_model_for_the_leaching_of_saline_soils_and_reclamation_improvement_amelioration_of_salty_areas_in_irrigated_lands_by_a_subsurface_drainage_system_includi

7. Appendix (computation of the non-linear reservoir reaction or response function Alfa From the parameters or characteristics or properties of a subsurface drainage system including the drainable porosity)

The parameters (characteristics) of an agricultural subsurface drainage system are illustrated in figure A.

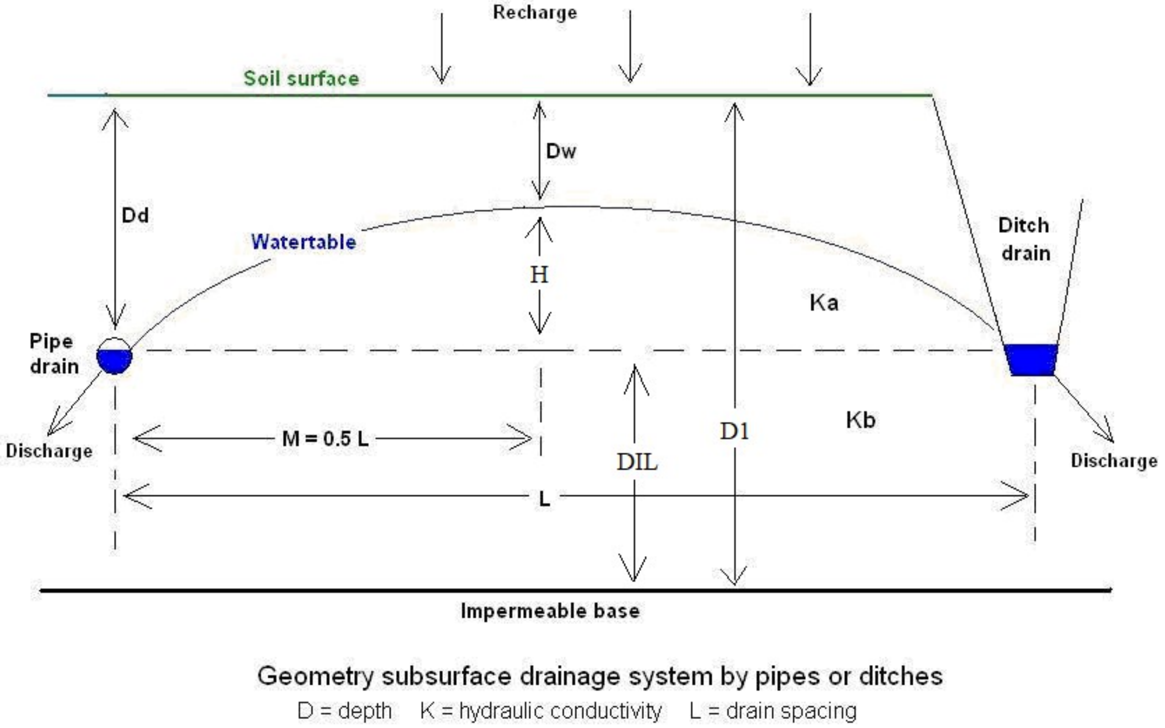


Figure A. *Parameters of of an agricultural subsurface drainage system*

In the situation of figure A, the steady state drainage equation of Hooghoudt is applicable [*Ref. 6*].:

$$Q = \frac{8Kb.De.H}{L^2} + \frac{4Ka.H^2}{L^2}$$

The height (H in m) of the water table midway between the drains above drain level equals Dd-Dw in figure A.
Ka and Kb = hydraulic conductivity above and below drain level respectively (m/day)
L = drain spacing (m)
De = equivalent depth of the impermeable layer below drain level. It depends on the actual depth DIL = D1 – Dd (see figure A) of the impermeable layer below drain level. The mathematical expression of De in terms of DIL is shown on the next page.
Q is expressed in m/day.
The drainable storage S of water midway between the drains equals S = Pd.H where Pd is the drainable porosity (in m/m) of the soil, also called effective porosity. In clay soils it normally varies between 2 and 4%, in loamy soils it may vary from 3 to 5% and in sandy loams it may range from 4 to 6% and in sandy soil from 5 to 10%

Writing $Q = \alpha.H$ we find:

$$\alpha = \frac{8Kb.De}{L^2} + \frac{4Ka.H}{L^2}$$

or:

$$\alpha = B + A.H$$

where:

$B = 8Kb.De / L^2$

$A = 4Ka / L^2$

yielding a reaction (response factor α) depending on the storage S (and therefore also on Q), so that we have a non linear reservoir.

In transient (un-steady state) the expressions of B and A need to be changed into [*Ref. 6*]:

$B = \pi^2.Kb.De / Pd.L^2$

$A = 0.5\ \pi^2.Ka / Pd.L^2$

From which follows: $De = 0.5B.Ka/A.Kb$

Equivalent depth De

Reference: W.H. van der Molen and J. Wesseling 1991. A solution in closed form and a series solution for the thickness of the equivalent layer in Hooghoudt's drain spacing formula. Agricultural Water Management 19, pp. 1-16

$$De = \frac{\pi\ L/8}{\ln (L/U) + F(x)}$$

where U = wet circumference of the drain (m) and F(x) is a function of

$x = 2\ \pi\ DIL / L$

When x>1 then:

$$F(x) = \frac{4e^{-2x}}{(1- e^{-2x})} + \frac{4e^{-6x}}{3(1- e^{-6x})} + \frac{4e^{-10x}}{5(1- e^{-10x})} + \ldots..$$

For x<=1:

$F(x) = \pi^2 / 4x + \ln (x/2\pi)$

Note.

For a half full pipe drain $U = \pi\ r$ with r = drain radius. For a ditch drain U equals bottom width + twice the length of the part of the sides that is under water.