Rainfall-runoff model with non-linear reservoir

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On website https://www.waterlog.info
Theory of the RainOff model to be found on https://www.waterlog.info/rainoff.htm

Introduction

The runoff from watershed (hydrologic catchment areas) resulting from rainfall can be simulated using the principles of the non-linear reservoir. The principles can also be applied to the discharge of agricultural drainage systems with pipe drains or ditches in response to recharge by rainfall or irrigation.

Hereunder, the basics of the linear reservoir are dealt with first, whereafter the non-linear reservoir is introduced. Finally, the use of the reservoir model in agricultural land drainage with pipes or ditches is illustrated.

The linear reservoir

The hydrology of a linear reservoir (figure 1) is governed by two equations.

1 - flow equation**:**

$$Q = A.S \qquad (1)$$

with Q in units [L/T], where L is length (e.g. mm) and T is time (e.g. hour, day)

2 - continuity or water balance equation**:**

$$R = Q + dS/dT, \text{ with units } [L/T] \qquad (2)$$

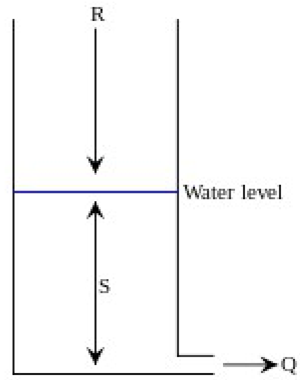


Figure 1 - A linear reservoir

Definition of symbols used:

Q is the *runoff* or *discharge*
R is the *effective rainfall* or *rainfall excess* or *recharge*
A is the constant *reaction factor* or *response factor* with unit [1/T]
S is the water storage with unit [L]
dS is a differential or small increment of S
dT is a differential or small increment of T

Runoff equation

A combination of the two previous equations results in a differential equation whose solution is:

$$Q2 = Q1 \exp \{ -A (T2 - T1) \} + R [1 - \exp \{ -A (T2 - T1) \}] \qquad (3)$$

This is the *runoff equation* or *discharge equation*, where Q1 and Q2 are the values of Q at time T1 and T2 respectively while T2−T1 (representing dT) is a small time step during which the recharge can be assumed constant.

Computing the total hydrograph

Provided the value of A is known, the *total hydrograph* can be obtained using a successive number of time steps and computing, with the *runoff equation*, the runoff at the end of each time step from the runoff at the end of the previous time step.

Unit hydrograph

The discharge may also be expressed as: $Q = -\, dS/dT$. Substituting herein the expression of Q in equation (1) gives the differential equation $dS/dT = A.S$, of which the solution is: $S = \exp(-\, A.t)$. Replacing herein S by Q/A according to equation (1), it is obtained that: $Q = A \exp(-\, A.t)$. This is called the instantaneous unit hydrograph (IUH) because the Q herein equals Q2 of the foregoing runoff equation using $R = 0$, and taking S as *unity* which makes Q1 equal to A according to equation (1).

Determining the response factor A

When the *response factor* A can be determined from the characteristics of the watershed (catchment area), the reservoir can be used as a *deterministic model* or *analytical model*. Otherwise, the factor A can be determined from a data record of rainfall and runoff using the method explained below under *non-linear reservoir*. With this method the reservoir can be used as a *black-box* model.

Conversions

1 mm/day corresponds to 10 m^3/day per ha of the watershed

1 l/s per ha corresponds to 8.64 mm/day or 86.4 m^3/day per ha

No recharge

During periods without rainfall or recharge, i.e. when *R* = 0, the runoff equation (3) reduces to

$$Q2 = Q1 \exp \{ - A (T2 - T1) \} \qquad (4)$$

or, using a *unit time step* (T2 − T1 = 1) and solving for Aq:

$$A = - \ln (Q2/Q1)$$

Hence, the reaction or response factor Aq can be determined from runoff or discharge measurements using *unit time steps* during dry spells (R=0).

Recharge

The recharge, also called *effective rainfall* or *rainfall excess*, can be modelled by a *pre-reservoir* (figure 2) giving the recharge as *overflow*. The pre-reservoir knows the following elements:

- a maximum storage (Sm) with unit length [L]
- an actual storage (Sa) with unit [L]
- a relative storage: Sr = Sa/Sm
- a maximum escape rate (Em) with units length/time [L/T]. It corresponds to the maximum rate of *evaporation* plus *percolation* to the groundwater, which will not take part in the runoff process (figure 5, 6)
- an actual escape rate: Ea = Sr.Em
- a storage deficiency: Sd = Sm + Ea − Sa

The recharge R during a unit time step (T2−T1=1) can be found from R = Rain − Sd (the overflow) when Rain-Sd>0, or R=0 when Rain-Sd<=0
The actual storage Sa_2 at the end of a *unit time step* is found as $Sa_2 = Sa_1 + Rain - R - Ea$, where Sa_1 is the actual storage at the start of the time step.

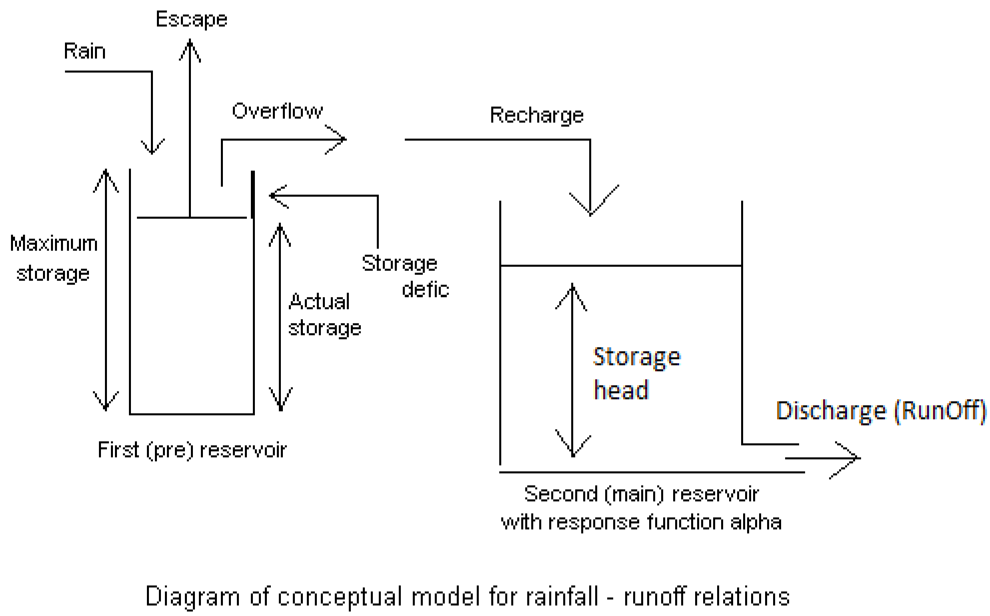


Figure 2. Illustrating the pre-reservoir determining the recharge into the linear reservoir

Figure 3 hereunder illustrates the escape factors evaporation, transpiration, and natural drainage to the underground. Figure 4 shows the rainfall and recharge data for s small valley.

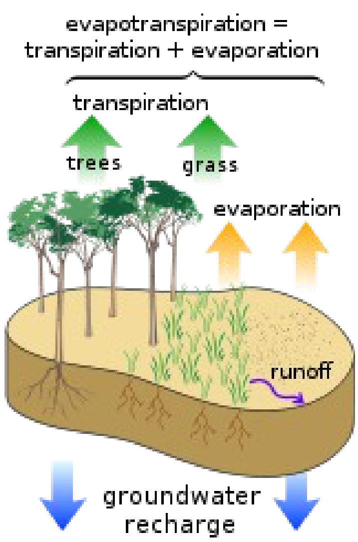


Figure 3. Escape factors evaporation and groundwater recharge.

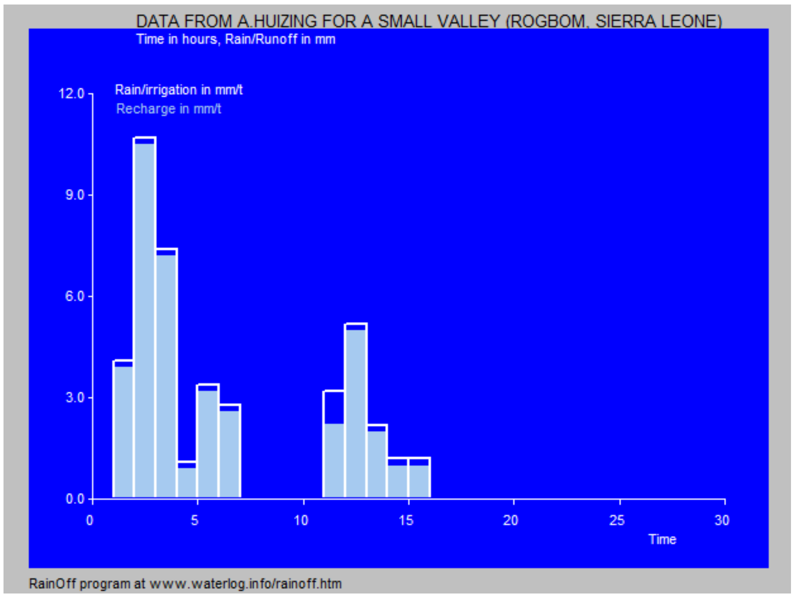


Figure 4. Rainfall converted into recharge for a small valley (Rogbom) in Sierra Leone.

The Curve Number Method (CN method) gives another way to calculate the recharge. The *initial abstraction* herein compares with Sm − Si, where Si is the initial value of Sa.

The non-linear reservoir

Contrary to the linear reservoir, the non linear reservoir has a reaction factor A that is not a constant, but it is a function of storage S (figure 1), because the number of outlets increases with increasing S.

In this figure it can be seen that the discharge increases exponentially when S increases.

The response factor here is not constant as for the linear reservoir and it will be called α instead of A

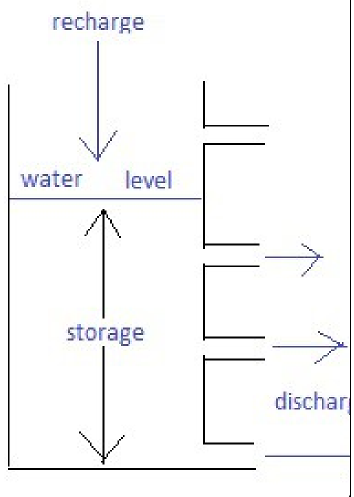
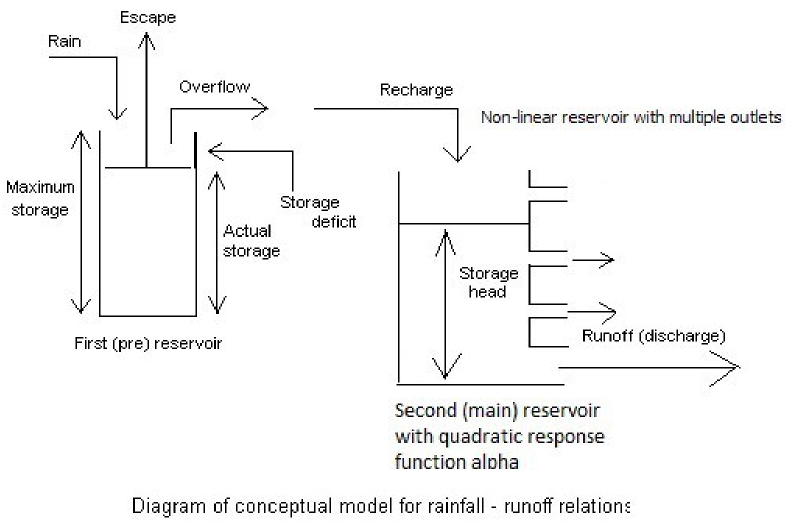


Figure 5. A non-linear reservoir

Figure 6. A pre-reservoir to find the effective recharge into the non-linear reservoir.



Instead of the constant reaction factor A in equation 1 and 3 for the linear reservoir, the reaction factor α is not a constant, but it depends on Q as in the following equation

$$\alpha = B.Q + C$$

Hence equation 3 changes into

$$Q = Q_1 \exp\{-(B.Q_1 + C)(T_2 - T_1)\} + R\,[1 - \exp\{-(B.Q_1 + C)(T_2 - T_1)\}]$$

Figure 7 shows the relation between α (Alpha) and Q for a small valley (Rogbom) in Sierra Leone. Here, it can be seen that α (Alpha) is a linear function of Q.

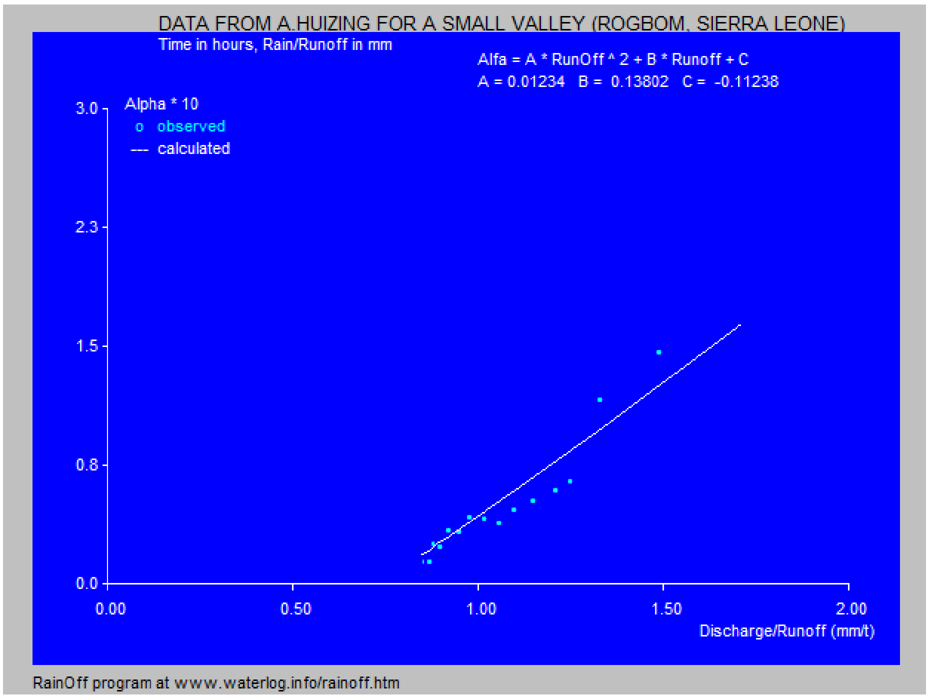


Figure 7. In a non-linear reservoir, the A (alpha) factor changes with increasing runoff (Q)

Figure 8 demonstrates the discharge calculated with a non-linear reservoir model and the fit to observed data on the basis of the recharge found in figure 4.

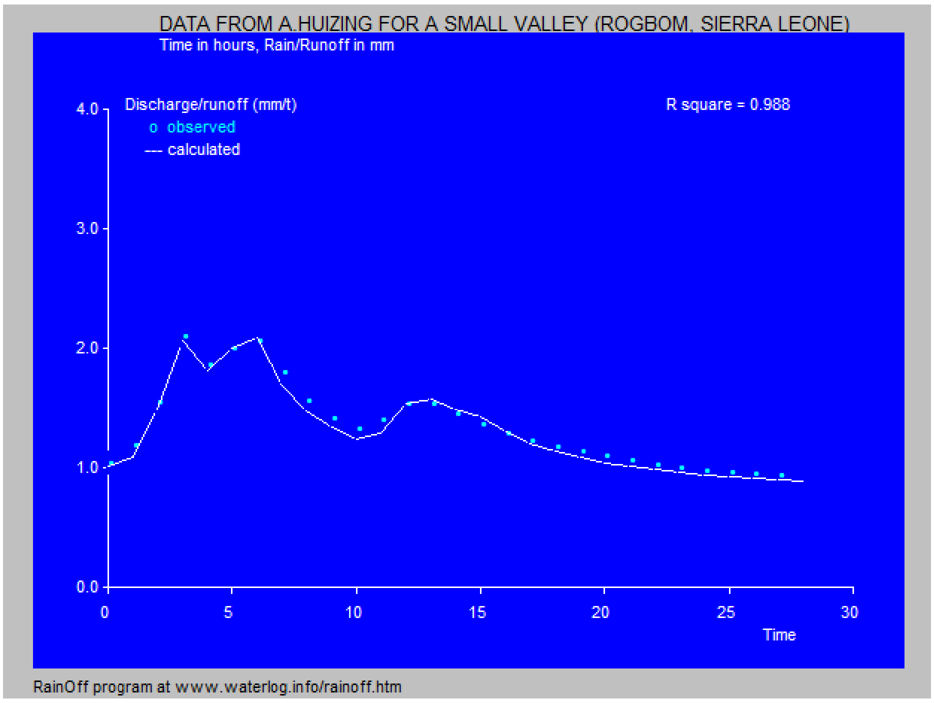
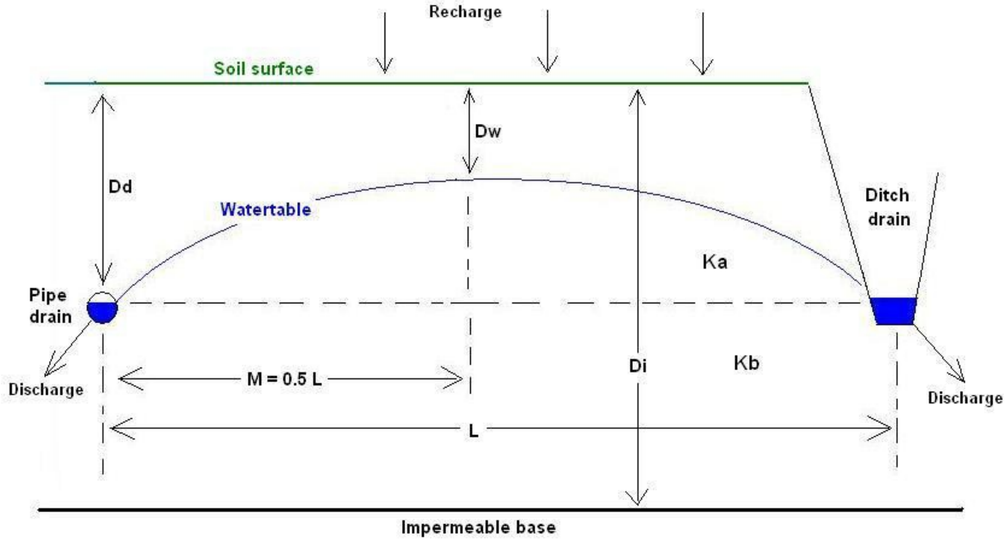


Figure 8. Calibration of observed and calculated (simulated) runoff in a small valley (Rogbom) in Sierra Leone using a non-linear reservoir model.

Agricultural land drainage



Geometry subsurface drainage system by pipes or ditches

D = depth K = hydraulic conductivity L = drain spacing

Figure 9.

In the situation of figure 9, the drainage equation of Hooghoudt is applicable.:

$$Q = \frac{8Kb.De.H}{L^2} + \frac{4Ka.H^2}{L^2}$$

The height (H in m) of the water table midway between the drains above drain level equals Dd-Dw in figure 9.

Ka and Kb = hydraulic conductivity above and below drain level respectively (m/day)

L = drain spacing (m)

De = equivalent depth of the impermeable layer below drain level. It depends on the actual depth Da = Di – Dd (see figure 9) of the impermeable layer below drain level. The mathematical expression of De in terms of Da is shown on the next page.

Q is expressed in m/day.

The drainable storage S of water midway between the drains equals S = Pd.H where Pd is the drainable porosity (in m/m) of the soil, also called effective porosity. In clay soils it normally varies between 2 and 4%, in loamy soils it may vary from 3 to 5% and in sandy loams it may range from 4 to 6% and in sandy soil from 5 to 10%

Writing Q = Aq.H = α.H we find

$$\alpha = \frac{8Kb.De}{L^2} + \frac{4Ka.H}{L^2}$$

or:

$$\alpha = B + A.H$$

where:

$$B = 8Kb.De / L^2$$

$$A = 4Ka / L^2$$

yielding a reaction (response factor α) depending on the storage S (and therefore also on Q), so that we have a non linear reservoir.

In transient (un-steady state) the expressions of B and A need to be changed into :

$$B = \pi^2.Kb.De / Pd.L^2$$

$$A = 0.5\, \pi^2.Ka / Pd.L^2$$

Equivalent depth De

Reference: W.H. van der Molen and J. Wesseling 1991. A solution in closed form and a series solution for the thickness of the equivalent layer in Hooghoudt's drain spacing formula. Agricultural Water Management 19, pp. 1-16

$$De = \frac{\pi L/8}{\ln(L/U) + F(x)}$$

where U = wet circumference of the drain (m) and F(x) is a function of

$$x = 2\pi Da / L$$

When x>1 then:

$$F(x) = \frac{4e^{-2x}}{(1-e^{-2x})} + \frac{4e^{-6x}}{3(1-e^{-6x})} + \frac{4e^{-10x}}{5(1-e^{-10x})} + \ldots$$

For x<=1:

$$F(x) = \pi^2 / 4x + \ln(x/2\pi)$$

Note.

For a half full pipe drain $U = \pi r$ with r = drain radius. For a ditch drain U equals bottom width + twice the length of the part of the sides that is under water.